CHAPTER 2

ELECTROSTATIC POTENTIAL AND CAPACITANCE

2.1 ELECTROSTATIC POTENTIAL AND POTENTIAL DIFFERENCE

Introduction. The electric field around a charge can be described in two ways :

(*i*) by electric field ($\vec{E}$), and

(*ii*) by electrostatic or electric potential (V).

The electric field $\vec{E}$ is a vector quantity, while electric potential is a scalar quantity. Both of these quantities are the characteristic properties of any point in a field and are inter-related.

1. *Develop the concepts of potential difference and electric potential. State and define their SI units.*

Potential difference. As shown in Fig. 2.1, consider a point charge $+q$ located at a point O. Let A and B be two points in its electric field. When a test charge q_0 is moved from A to B, a work W_{AB} has to be done in moving against the repulsive force exerted by the charge $+q$. We then calculate the potential difference between points A and B by the equation :

$$V = V_B - V_A = \frac{W_{AB}}{q_0} \quad ...(2.1)$$

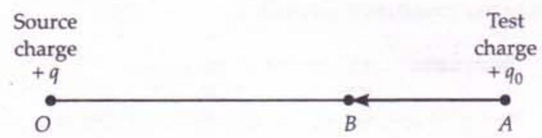


Fig. 2.1 To define potential difference.

So the ***potential difference*** *between two points in an electric field may be defined as the amount of work done in moving a unit positive charge from one point to the other against the electrostatic forces.*

In the above definition, we have assumed that the test charge is so small that it does not disturb the distribution of the source charge. Secondly, we just apply so much external force on the test charge that it just balances the repulsive electric force on it and hence does not produce any acceleration in it.

SI unit of potential difference is volt (V). It has been named after the Italian scientist *Alessandro Volta*.

$$1 \text{ volt} = \frac{1 \text{ joule}}{1 \text{ coulomb}}$$

or $$1\text{ V} = 1\text{ Nm C}^{-1} = 1\text{ JC}^{-1}$$

Hence the ***potential difference*** between two points in an electric field is said to be ***1 volt*** *if 1 joule of work has to be done in moving a positive charge of 1 coulomb from one point to the other against the electrostatic forces.*

Electric potential. The electric potential at a point located far away from a charge is taken to be *zero*.

In Fig. 2.1, if the point A lies at infinity, then $V_A = 0$, so that

$$V = V_B = \frac{W}{q_0}$$

where W is the amount of work done in moving the test charge q_0 from infinity to the point B and V_B refers to the potential at point B.

So the ***electric potential*** *at a point in an electric field is the amount of work done in moving a unit positive charge from infinity to that point against the electrostatic forces.*

$$\text{Electric potential} = \frac{\text{Work done}}{\text{Charge}}$$

SI unit of electric potential is volt (V). The ***electric potential*** *at a point in an electric field is said to be* ***1 volt*** *if one joule of work has to be done in moving a positive charge of 1 coulomb from infinity to that point against the electrostatic forces.*

2.2 ELECTRIC POTENTIAL DUE TO A POINT CHARGE

2. *Derive an expression for the electric potential at a distance r from a point charge q. What is the nature of this potential ?*

Electric potential due to a point charge. Consider a positive point charge q placed at the origin O. We wish to calculate its electric potential at a point P at distance r from it, as shown in Fig. 2.2. By definition, the electric potential at point P will be equal to the amount of work done in bringing a unit positive charge from infinity to the point P.

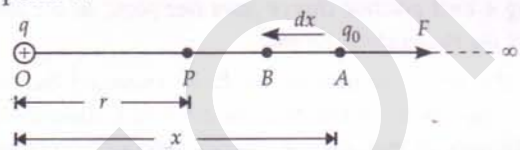


Fig. 2.2 Electric potential due to a point charge.

Suppose a test charge q_0 is placed at point A at distance x from O. By Coulomb's law, the electrostatic force acting on charge q_0 is

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qq_0}{x^2}$$

The force $\vec{F}$ acts away from the charge q. The small work done in moving the test charge q_0 from A to B through small displacement $\vec{dx}$ against the electrostatic force is

$$dW = \vec{F} \cdot \vec{dx} = Fdx \cos 180° = -Fdx$$

The total work done in moving the charge q_0 from infinity to the point P will be

$$W = \int dW = -\int_\infty^r Fdx = -\int_\infty^r \frac{1}{4\pi\varepsilon_0} \cdot \frac{qq_0}{x^2}\, dx$$

$$= -\frac{qq_0}{4\pi\varepsilon_0} \int_\infty^r x^{-2}\, dx = -\frac{qq_0}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]_\infty^r$$

$$= \frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qq_0}{r}.$$

Hence the work done in moving a unit test charge from infinity to the point P, or the electric potential at point P is

$$V = \frac{W}{q_0} \quad \text{or} \quad V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

Clearly, $V \propto 1/r$. *Thus the electric potential due to a point charge is spherically symmetric* as it depends only on the distance of the observation point from the charge and not on the direction of that point with respect to the point charge. Moreover, we note that the potential at infinity is zero.

Figure 2.3 shows the variation of electrostatic potential ($V \propto 1/r$) and the electrostatic field ($E \propto 1/r^2$) with distance r from a charge q.

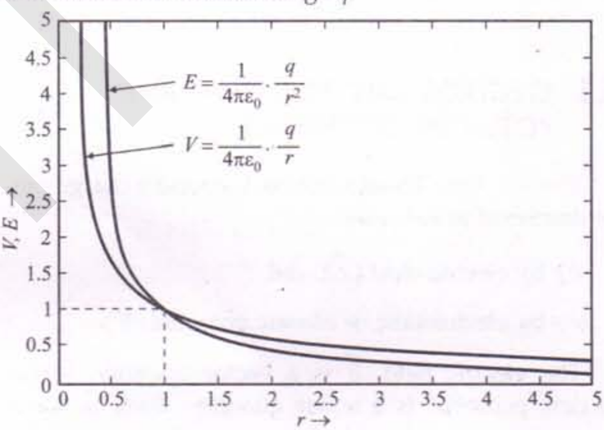


Fig. 2.3 Variation of potential V and field E with r from a point charge q.

2.3 ELECTRIC POTENTIAL DUE TO A DIPOLE

3. *Derive an expression for the potential at a point along the axial line of a short dipole.*

Electric potential at an axial point of a dipole. As shown in Fig. 2.4, consider an electric dipole consisting of two point charges $-q$ and $+q$ and separated by distance $2a$. Let P be a point on the axis of the dipole at a distance r from its centre O.

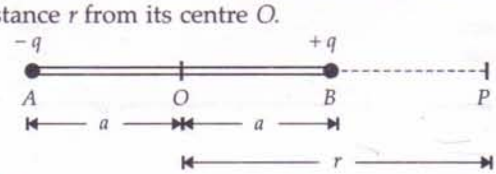


Fig. 2.4 Potential at an axial point of a dipole.

Electric potential at point P due to the dipole is

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{-q}{AP} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{BP}$$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r+a} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r-a}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r-a} - \frac{1}{r+a}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+a)-(r-a)}{r^2-a^2}\right] = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\times 2a}{r^2-a^2}$$

or $$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2-a^2} \qquad [\because p = q\times 2a]$$

For a short dipole, $a^2 << r^2$, so $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2}$.

4. *Show mathematically that the potential at a point on the equatorial line of an electric dipole is zero.*

Electric potential at an equatorial point of a dipole. As shown in Fig. 2.5, consider an electric dipole consisting of charges $-q$ and $+q$ and separated by distance $2a$. Let P be a point on the perpendicular bisector of the dipole at distance r from its centre O.

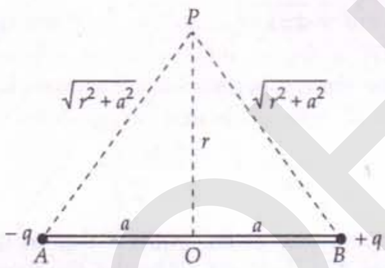


Fig. 2.5 Potential at an equatorial point of a dipole.

Electric potential at point P due to the dipole is

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{-q}{AP} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{BP}$$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{\sqrt{r^2+a^2}} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{\sqrt{r^2+a^2}} = 0.$$

5. *Derive an expression for the electric potential at any general point at distance r from the centre of a dipole.*

Electric potential at any general point due to a dipole. Consider an electric dipole consisting of two point charges $-q$ and $+q$ and separated by distance $2a$, as shown in Fig. 2.6. We wish to determine the potential at a point P at a distance r from the centre O, the direction OP making an angle θ with dipole moment $\vec{p}$.

Let $AP = r_1$ and $BP = r_2$.

Net potential at point P due to the dipole is

$$V = V_1 + V_2$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{-q}{r_1} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r_2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r_1 - r_2}{r_1 r_2}\right]$$

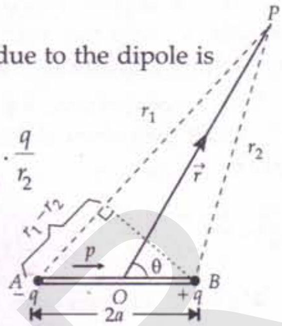


Fig. 2.6

If the point P lies far away from the dipole, then

$$r_1 - r_2 \simeq AB\cos\theta = 2a\cos\theta \quad\text{and}\quad r_1 r_2 \simeq r^2$$

$$\therefore \qquad V = \frac{q}{4\pi\varepsilon_0}\cdot\frac{2a\cos\theta}{r^2}$$

or $$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p\cos\theta}{r^2}$$

or $$V = \frac{1}{4\pi\varepsilon_0}\,\frac{\vec{p}\cdot\vec{r}}{r^3} = \frac{1}{4\pi\varepsilon_0}\,\frac{\vec{p}\cdot\hat{r}}{r^2}$$

Here $p = q\times 2a$, is the dipole moment and $\hat{r} = \vec{r}/r$, is a unit vector along the position vector $\vec{OP} = \vec{r}$.

Special Cases

(*i*) When the point P lies on the *axial line* of the dipole, $\theta = 0°$ or $180°$, and

$$V = \pm\frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2}$$

i.e., the potential has greatest positive or the greatest negative value.

(*ii*) When the point P lies on the *equatorial line* of the dipole, $\theta = 90°$, and $V = 0$, *i.e.*, the potential at any point on the equatorial line of the dipole is zero. However, the electric field at such points is non-zero.

6. *Give the contrasting features of electric potential of a dipole from that due to a single charge.*

Differences between electric potentials of a dipole and a single charge.

1. The potential due to a dipole depends not only on distance r but also on the angle between the position vector $\vec{r}$ of the observation point and the dipole moment vector $\vec{p}$. The potential due to a single charge depends only on r.

2. *The potential due to a dipole is cylindrically symmetric* about the dipole axis. If we rotate the observation point

P about the dipole axis (keeping r and θ fixed), the potential V does not change. The potential due to a *single charge is spherically symmetric.*

3. At large distance, the dipole potential falls off as $1/r^2$ while the potential due to a single charge falls off as $1/r$.

2.4 ELECTRIC POTENTIAL DUE TO A SYSTEM OF CHARGES

7. *Derive an expression for the electric potential at a point due to a group of N point charges.*

Electric potential due to a group of point charges. As shown in Fig. 2.7, suppose N point charges $q_1, q_2, q_3, \ldots, q_N$ lie at distances $r_1, r_2, r_3, \ldots, r_N$ from a point P.

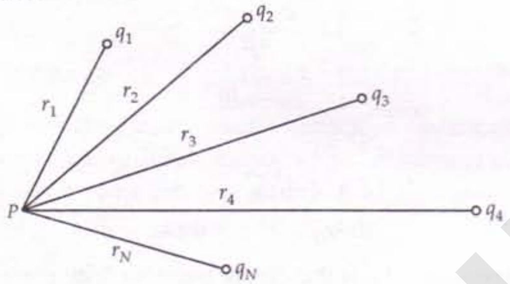


Fig. 2.7 Potential at a point due to a system of N point charges.

Electric potential at point P due to charge q_1 is

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_1}$$

Similarly, electric potentials at point P due to other charges will be

$$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_2},\ V_3 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_3}{r_3},\ \ldots,\ V_N = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_N}{r_N}$$

As electric potential is a scalar quantity, so the total potential at point P will be equal to the algebraic sum of all the individual potentials, *i.e.*,

$$V = V_1 + V_2 + V_3 + \ldots + V_N$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots + \frac{q_N}{r_N}\right]$$

or

$$V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i}$$

If $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots, \vec{r}_N$ are the position vectors of the N point charges, the electric potential at a point whose position vector is $\vec{r}$, would be

$$V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{|\vec{r} - \vec{r}_i|}.$$

2.5 ELECTRIC POTENTIAL DUE TO A CONTINUOUS CHARGE DISTRIBUTION

8. *Deduce an expression for the potential at a point due to a continuous charge distribution. Hence write the expression for the electric potential due to a general source.*

Electric potential due to a continuous charge distribution. We can imagine that a continuous charge distribution consists of a number of small charge elements located at positions $\vec{r}_i$. If $\vec{r}$ is the position vector of point P, then the electric potential at point P due to the continuous charge distribution can be written as

$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{|\vec{r} - \vec{r}_i|}$$

When the charge is distributed continuously in a volume V, $dq = \rho\, dV$, where ρ is volume charge density. The potential at point P due to the volume charge distribution will be

$$V_V = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho\, dV}{|\vec{r} - \vec{r}_i|}$$

When the charge is distributed continuously over an area S, $dq = \sigma\, dS$ where σ is surface charge density.

$$\therefore \quad V_S = \frac{1}{4\pi\varepsilon_0} \int_S \frac{\sigma\, dS}{|\vec{r} - \vec{r}_i|}$$

When the charge is distributed uniformly along a line L, $dq = \lambda\, dL$, where λ is line charge density.

$$\therefore \quad V_L = \frac{1}{4\pi\varepsilon_0} \int_L \frac{\lambda\, dL}{|\vec{r} - \vec{r}_i|}$$

The net potential at the point P due to the continuous charge distribution will be the algebraic sum of the above potentials.

$$V_{cont} = V_V + V_S + V_L$$

or

$$V_{cont} = \frac{1}{4\pi\varepsilon_0}\left[\int_V \frac{\rho\, dV}{|\vec{r} - \vec{r}_i|} + \int_S \frac{\sigma\, dS}{|\vec{r} - \vec{r}_i|} + \int_L \frac{\lambda\, dL}{|\vec{r} - \vec{r}_i|}\right]$$

Electric potential due to a general source. The potential due to a general source charge distribution, which consists of continuous as well as discrete point charges, can be written as

$$V = V_{cont} + V_{discrete}$$

or

$$V = \frac{1}{4\pi\varepsilon_0}\left[\int_V \frac{\rho\, dV}{|\vec{r} - \vec{r}_i|} + \int_S \frac{\sigma\, dS}{|\vec{r} - \vec{r}_i|} + \int_L \frac{\lambda\, dL}{|\vec{r} - \vec{r}_i|} + \sum_{\text{All point charges}} \frac{q_i}{|\vec{r} - \vec{r}_i|}\right]$$

2.6 ELECTRIC POTENTIAL DUE TO A UNIFORMLY CHARGED THIN SPHERICAL SHELL

9. *Write expression for the electric potential due to a uniformly charged spherical shell at a point (i) outside the shell, (ii) on the shell and (iii) inside the shell.*

Electric potential due to uniformly charged thin spherical shell. Consider a uniformly charged spherical shell of radius R and carrying charge q. We wish to calculate its potential at point P at distance r from its centre O, as shown in Fig. 2.8.

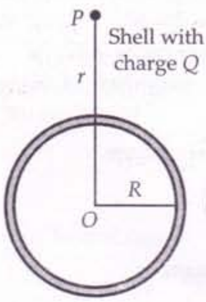


Fig. 2.8 Potential due to a spherical shell.

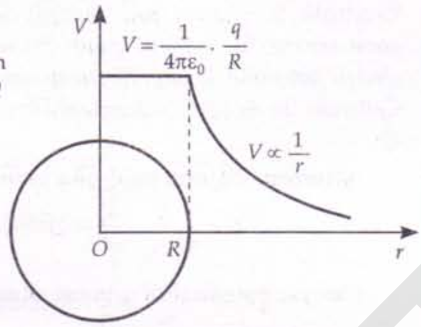


Fig. 2.9 Variation of potential due to charged shell with distance r from its centre.

(*i*) *When the point P lies outside the shell.* We know that for a uniformly charged spherical shell, the electric field outside the shell is as if the entire charge is concentrated at the centre. Hence electric potential at an outside point is equal to that of a point charge located at the centre, which is given by

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} \qquad \text{[For } r > R\text{]}$$

(*ii*) *When point P lies on the surface of the shell.* Here $r = R$. Hence the potential on the surface of the shell is

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R} \qquad \text{[For } r = R\text{]}$$

(*iii*) *When point P lies inside the shell.* The electric field at any point inside the shell is zero. Hence electric potential due to a uniformly charged spherical shell is constant everywhere inside the shell and its value is equal to that on the surface. Thus,

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R} \qquad \text{[For } r < R\text{]}$$

Figure 2.9 shows the variation of the potential V due to a uniformly charged spherical shell with distance r measured from the centre of the shell. Note that V is constant $(= q / 4\pi\varepsilon_0 R)$ from $r = 0$ to $r = R$ along a horizontal line and thereafter $V \propto 1/r$ for points outside the shell.

For Your Knowledge

- Electric potential is a scalar quantity while potential gradient is a vector quantity.
- The electric potential near an *isolated positive charge is positive* because work has to be done by an external agent to push a positive charge in, from infinity.
- The electric potential near an *isolated negative charge is negative* because the positive test charge is attracted by the negative charge.
- The electric potential due to a charge q at its own location is not defined – it is infinite.
- Because of arbitrary choice of the reference point, the electric potential at a point is arbitrary to within an additive constant. But it is immaterial because it is the potential difference between two points which is physically significant.
- For defining electric potential at any point, generally a point far away from the source charges is taken as the reference point. Such a point is assumed to be at infinity.
- As the electrostatic force is a conservative force, so the work done in moving a unit positive charge from one point to another or the potential difference between two points does not depend on the path along which the test charge is moved.

Examples based on Electric Potential

Formulae Used

1. Potential difference $= \frac{\text{Work done}}{\text{Charge}}$ or $V = \frac{W}{q}$
2. Electric potential due to a point charge q at distance r from it,
$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$
3. Electric potential at a point due to N point charges,
$$V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i}$$
4. Electric potential at a point due to a dipole,
$$V = \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{p} \cdot \vec{r}}{r^3}$$

Units Used

Charge q is in coulomb, distance r in metre, work done W in joule and potential difference V in volt.

Example 1. *If 100 J of work has to be done in moving an electric charge of 4C from a place, where potential is – 10 V to another place, where potential is V volt, find the value of V.*

Solution. Here $W_{AB} = 100$ J, $q_0 = 4$C, $V_A = -10$ V, $V_B = V$.

As $$V_B - V_A = \frac{W_{AB}}{q_0}$$

$$\therefore \quad V - (-10) = \frac{100}{4} = 25$$

or $$V = 25 - 10 = \mathbf{15\ V.}$$

Example 2. *Determine the electric potential at the surface of a gold nucleus. The radius is 6.6×10^{-15} m and the atomic number Z = 79. Given charge on a proton $= 1.6 \times 10^{-19}$ C.* [Himachal 96]

Solution. As nucleus is spherical, it behaves like a point charge for external points.

Here $q = ne = 79 \times 1.6 \times 10^{-19}$ C,

$r = 6.6 \times 10^{-15}$ m

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = \frac{9 \times 10^9 \times 79 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-15}} \text{ V}$$

$$= \mathbf{1.7 \times 10^7\ V.}$$

Example 3. *(i) Calculate the potential at a point P due to a charge of 4×10^{-7} C located 9 cm away. (ii) Hence obtain the work done in bringing a charge of 2×10^{-9} C from infinity to the point P. Does the answer depend on the path along which the charge is brought ?* [NCERT]

Solution. (*i*) Here $q = 4 \times 10^{-7}$ C, $r = 9$ cm $= 0.09$ m

Electric potential at point *P* is

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = 9 \times 10^9 \times \frac{4 \times 10^{-7}}{0.09} = \mathbf{4 \times 10^4\ V.}$$

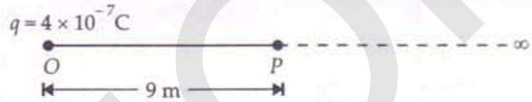


Fig. 2.10

(*ii*) By definition, electric potential at point *P* is equal to the work done in bringing a unit positive charge from infinity to the point *P*. Hence the workdone in bringing a charge of 2×10^{-9} C from infinity to the point *P* is

$$W = q_0 V = 2 \times 10^{-9} \times 4 \times 10^4 = \mathbf{8 \times 10^{-5}\ J}$$

No, the answer does not depend on the path along which the charge is brought.

Example 4. *A metal wire is bent in a circle of radius 10 cm. It is given a charge of 200 μC which spreads on it uniformly. Calculate the electric potential at its centre.* [CBSE OD 95C]

Solution. Here $q = 200\ \mu\text{C} = 2 \times 10^{-4}$ C,

$r = 10$ cm $= 0.10$ m

We can consider the circular wire to be made of a large number of elementary charges *dq*. Potential due to one such elementary charge *dq* at the centre,

$$dV = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{r}$$

Total potential at the centre of the circular wire,

$$V = \Sigma\, dV = \Sigma \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0\, r} \Sigma\, dq$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = \frac{9 \times 10^9 \times 2 \times 10^{-4}}{0.10} = \mathbf{18 \times 10^6\ V.}$$

Example 5. *Electric field intensity at point 'B' due to a point charge 'Q' kept at point 'A' is 24 NC^{-1} and the electric potential at point 'B' due to same charge is 12 JC^{-1}. Calculate the distance AB and also the magnitude of charge Q.* [CBSE OD 03C]

Solution. Electric field of a point charge,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} = 24 \text{ NC}^{-1}$$

Electric potential of a point charge,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r} = 12 \text{ JC}^{-1}$$

The distance *AB* is given by

$$r = \frac{V}{E} = \frac{12}{24} = \mathbf{0.5\ m}$$

The magnitude of the charge,

$$Q = 4\pi\varepsilon_0 Vr = \frac{1}{9 \times 10^9} \times 12 \times 0.5 = \mathbf{0.667 \times 10^{-9}\ C}$$

Example 6. *To what potential we must charge an insulated sphere of radius 14 cm so that the surface charge density is equal to 1 μ Cm^{-2} ?*

Solution. Here $r = 14$ cm $= 14 \times 10^{-2}$ m,

$\sigma = 1\ \mu\ \text{Cm}^{-2} = 10^{-6}\ \text{Cm}^{-2}$

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4\pi r^2 \sigma}{r} = \frac{1}{4\pi\varepsilon_0} \cdot 4\pi r\sigma$$

$$= 9 \times 10^9 \times 4 \times \frac{22}{7} \times 14 \times 10^{-2} \times 10^{-6} \text{ V}$$

$$= \mathbf{15840\ V.}$$

Example 7. *A charge of 24 μC is given to a hollow metallic sphere of radius 0.2 m. Find the potential* [CBSE D 95]

(*i*) *at the surface of the sphere, and*

(*ii*) *at a distance of 0.1 cm from the centre of the sphere.*

Solution. (*i*) $q = 24\ \mu\text{C} = 24 \times 10^{-6}$ C, $R = 0.2$ m

Potential at the surface of the sphere is

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R} = \frac{9 \times 10^9 \times 24 \times 10^{-6}}{0.2} \text{ V} = \mathbf{1.08 \times 10^6\ V.}$$

(*ii*) As potential at any point inside the sphere

= Potential on the surface

$\therefore$ Potential at a distance of 0.1 cm from the centre

$= \mathbf{1.08 \times 10^6\ V.}$

Example 8. *Twenty seven drops of same size are charged at 220 V each. They coalesce to form a bigger drop. Calculate the potential of the bigger drop.* [Punjab 01]

Solution. Let radius of each small drop = r

Radius of large drop = R

Then $\frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$

or $R = 3r$

Potential of each small drop,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

$\therefore$ Total charge on 27 drops,

$$Q = 27\,q = 27 \times 4\pi\varepsilon_0\, r\, V$$

Potential of large drop,

$$V' = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{27 \times 4\pi\varepsilon_0\, r\, V}{3r}$$

$$= 9\,V = 9 \times 220 = \mathbf{1980\ V.}$$

Example 9. *Two charges 3×10^{-8} C and -2×10^{-8} C are located 15 cm apart. At what point on the line joining the two charges is the electric potential zero ? Take the potential at infinity to be zero.* [NCERT]

Solution. As shown in Fig. 2.11, suppose the two point charges are placed on X-axis with the positive charge located on the origin O.

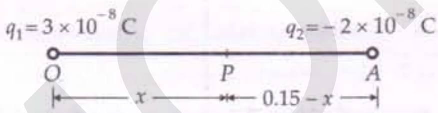


Fig. 2.11 Zero of electric potential for two charges.

Let the potential be zero at the point P and $OP = x$. For $x < 0$ (*i.e.*, to the left of O), the potentials of the two charges cannot add up to zero. Clearly, x must be positive. If x lies between O and A, then

$$V_1 + V_2 = 0$$

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{x} + \frac{q_2}{0.15 - x}\right] = 0$$

or $$9 \times 10^9 \left[\frac{3 \times 10^{-8}}{x} - \frac{2 \times 10^{-8}}{0.15 - x}\right] = 0$$

or $$\frac{3}{x} - \frac{2}{0.15 - x} = 0$$

which gives $x = 0.09$ m $=$ **9 cm**

The other possibility is that x may also lie on OA produced, as shown in Fig. 2.12.

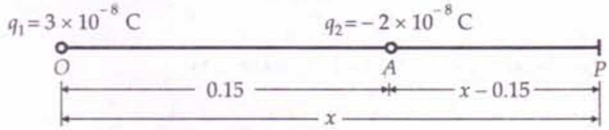


Fig. 2.12

As $$V_1 + V_2 = 0$$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\left[\frac{3 \times 10^{-8}}{x} - \frac{2 \times 10^{-8}}{x - 0.15}\right] = 0$$

which gives $x = 0.45$ m $=$ **45 cm**

Thus the electric potential is zero at 9 cm and 45 cm away from the positive charge on the side of the negative charge.

Example 10. *Calculate the electric potential at the centre of a square of side $\sqrt{2}$ m, having charges 100 μC, − 50 μC, 20 μC, and − 60 μC at the four corners of the square.* [CBSE OD 06C]

Solution. Diagonal of the square

$$= \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = 2\ \text{m}$$

Distance of each charge from the centre of the square is

$$r = \text{Half diagonal} = 1\ \text{m}$$

$\therefore$ Potential at the centre of the square is

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right]$$

$$V = 9 \times 10^9 \left[\frac{100 \times 10^{-6}}{1} - \frac{50 \times 10^{-6}}{1} + \frac{20 \times 10^{-6}}{1} - \frac{60 \times 10^{-6}}{1}\right]$$

$$= 9 \times 10^9 \times 10^{-6} \times 10 = \mathbf{9 \times 10^4\ V.}$$

Example 11. *Four charges + q, + q, − q and − q are placed respectively at the corners A, B, C and D of a square of side 'a' arranged in the given order. Calculate the electric potential at the centre O. If E and F are the midpoints of sides BC and CD respectively, what will be the work done in carrying a charge 'e' from O to E and from O to F ?*

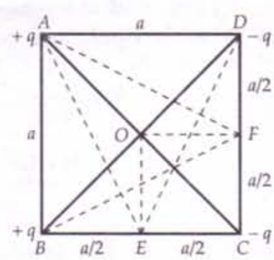


Fig. 2.13

Solution. Let $OA = OB = OC = OD = r$.

Then the potential at the centre O is

$$V_O = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r}+\frac{q}{r}-\frac{q}{r}-\frac{q}{r}\right]=0$$

Again, the potential at point E is

$$V_E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{AE}+\frac{q}{BE}-\frac{q}{CE}-\frac{q}{DE}\right]=0$$

$[\because AE = DE, BE = CE]$

Now, $AF = BF = \sqrt{a^2+\left(\frac{a}{2}\right)^2} = \frac{\sqrt{5}a}{2}$

$\therefore$ The potential at point F is

$$V_F = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{AF}+\frac{q}{BF}-\frac{q}{CF}-\frac{q}{DF}\right]$$

$$= \frac{2q}{4\pi\varepsilon_0}\left[\frac{1}{AF}-\frac{1}{CF}\right] \quad [\because AF = BF, CF = DF]$$

$$= \frac{2q}{4\pi\varepsilon_0}\left[\frac{2}{\sqrt{5}a}-\frac{2}{a}\right] = \frac{q}{\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{5}}-1\right)$$

Work done in moving the charge 'e' from O to E is

$$W = e\,[V_E - V_O] = e \times 0 = \mathbf{0}$$

Work done in moving the charge 'e' from O to F is

$$W = e[V_F - V_O] = e\left[\frac{q}{\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{5}}-1\right)-0\right]$$

$$= \frac{qe}{\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{5}}-1\right).$$

Example 12. *A short electric dipole has dipole moment of* 4×10^{-9} *Cm Determine the electric potential due to the dipole at a point distant* 0.3 *m from the centre of the dipole situated (a) on the axial line (b) on equatorial line and (c) on a line making an angle of* $60°$ *with the dipole axis.*

Solution. Here $p = 4 \times 10^{-9}$ Cm, $r = 0.3$ m.

(*a*) Potential at a point on the axial line is

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2} = \frac{9\times10^9\times4\times10^{-9}}{(0.3)^2} = \mathbf{400\ V.}$$

(*b*) Potential at a point on the equatorial line = **0**.

(*c*) Potential at a point on a line that makes an angle of $60°$ with dipole axis is

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p\cos\theta}{r^2}$$

$$= \frac{9\times10^9\times4\times10^{-9}\cos 60°}{(0.3)^2} = \mathbf{200\ V.}$$

Example 13. *Two point charges of* $+3\,\mu$C *and* $-3\,\mu$C *are placed* 2×10^{-3} *m apart from each other. Calculate (i) electric field and electric potential at a distance of* 0.6 *m from the dipole in broad-side-on position (ii) electric field and electric potential at the same point after rotating the dipole through* $90°$.

Solution. Dipole moment,

$$p = q \times 2l = 3\times10^{-6}\times2\times10^{-3} = 6\times10^{-9}\,\text{Cm}$$

(*i*) Electric field in broad-side-on position is

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^3} = \frac{9\times10^9\times6\times10^{-9}}{(0.6)^3} = \mathbf{250\ NC^{-1}}$$

Electric potential in broad-side-on position, $V = 0$.

(*ii*) When the dipole is rotated through $90°$, the same point is now in end-on-position with respect to the dipole.

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3} = \mathbf{500\ NC^{-1}}$$

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2} = \frac{9\times10^9\times6\times10^{-9}}{(0.6)^2} = \mathbf{150\ V.}$$

Example 14. *Two charges* $-q$ *and* $+q$ *are located at points* $A(0,0,-a)$ *and* $B(0,0,+a)$ *respectively. How much work is done in moving a test charge from point* $P(7,0,0)$ *to* $Q(-3,0,0)$? [CBSE D 09]

Solution. Points P and Q are located on the equatorial line of the electric dipole and potential of the dipole at any equatorial point is zero.

$\therefore$ Work done in moving a test charge q_0 from P to Q,

$$W = q_0(V_Q - V_P) = q_0(0-0) = \mathbf{0}.$$

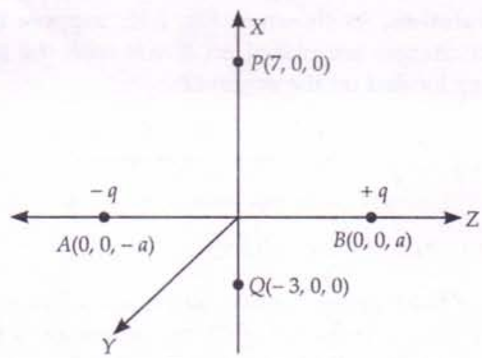


Fig. 2.14

Problems For Practice

1. The work done in moving a charge of 3 C between two points is 6 J. What is the potential difference between the two points ? (Ans. 2 V)
2. The electric potential at 0.9 m from a point charge is + 50 V. What is the magnitude and sign of the charge ? **[CBSE D 95C]** (Ans. 5×10^{-9} C, positive)

3. The electric field at a point due to a point charge is $20\ NC^{-1}$ and the electric potential at that point is $10\ JC^{-1}$. Calculate the distance of the point from the charge and the magnitude of the charge. **[CBSE D 06]**

(Ans. 0.5 m, 0.55×10^{-9}C)

4. Two points A and B are located in diametrically opposite directions of a point charge of $+2\ \mu C$ at distances 2.0 m and 1.0 m respectively from it. Determine the potential difference $V_A - V_B$.

(Ans. -9×10^3 V)

5. A hollow metal sphere is charged with $0.4\ \mu C$ of charge and has a radius of 0.1 m. Find the potential (*i*) at the surface (*ii*) inside the sphere (*iii*) at a distance of 0.6 m from the centre. The sphere is placed in air. (Ans. 36 kV, 36 kV, 6 kV)

6. Two point charges of $+10\ \mu C$ and $+20\ \mu C$ are placed in free space 2 cm apart. Find the electric potential at the middle point of the line joining the two charges. (Ans. 27 MV)

7. Two point charges q and $-2q$ are kept 'd' distance apart. Find the location of the point relative to charge 'q' at which potential due to this system of charges is zero. **[CBSE OD 14C]**

(Ans. At distance $d/3$ from charge q)

8. Two point charges, one of $+100\ \mu C$ and another of $-400\ \mu C$, are kept 30 cm apart. Find the points of zero potential on the line joining the two charges (assume the potential at infinity to be zero).

(Ans. 6 cm from $+100\ \mu C$ charge)

9. A charge $q = +1\ \mu C$ is held at O between the points A and B such that $AO = 2$ m and $BO = 1$ m, as shown in Fig. 2.15(*a*). Calculate the potential difference $(V_A - V_B)$. What will be the value of the potential difference $(V_A - V_B)$ if position of B is changed as shown in Fig. 2.15(*b*) ? (Ans. -4500 V, -4500 V)

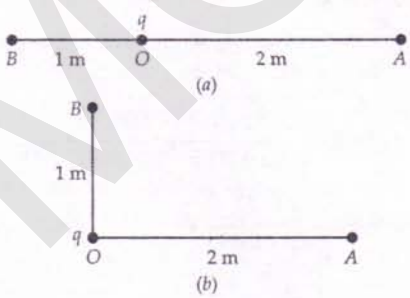


Fig. 2.15

10. Two small spheres of radius 'a' each carrying charges $+q$ and $-q$ are placed at points A and B, distance 'd' apart. Calculate the potential difference between points A and B. (Ans. $2q/4\pi\varepsilon_0 d$)

11. The sides of rectangle $ABCD$ are 15 cm and 5 cm, as shown in Fig. 2.16. Point charges of $-5\ \mu C$ and $+2\ \mu C$ are placed at the vertices B and D respectively. Calculate electric potentials at the vertices A and C. Also calculate the work done in carrying a charge of $3\ \mu C$ from A to C. (Ans. 2.52 J)

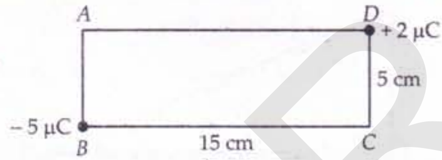


Fig. 2.16

12. Charges of 2.0×10^{-6} C and 1.0×10^{-6} C are placed at the corners A and B of a square of side 5.0 cm as shown in Fig. 2.17. How much work will be done in moving a charge of 1.0×10^{-6}C from C to D against the electric field ? (Ans. 0.053 J)

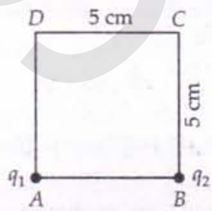


Fig. 2.17

13. Calculate the potential at the centre of a square $ABCD$ of each side $\sqrt{2}$ m due to charges 2, -2, -3 and $6\ \mu C$ at four corners of it. **[Haryana 97]**

(Ans. 2.7×10^4 V)

14. Charges of $+1.0 \times 10^{-11}$C, -2.0×10^{-11} C, $+1.0 \times 10^{-11}$ C are placed respectively at the corners B, C and D of a rectangle $ABCD$. Determine the potential at the corner A. Given $AB = 4$ cm and $BC = 3$ cm.

(Ans. 1.65 V)

15. $ABCD$ is a square of side 0.2 m. Charges of 2×10^{-9}, 4×10^{-9}, 8×10^{-9} C are placed at the corners A, B and C respectively. Calculate the work required to transfer a charge of 2×10^{-9} C from D to the centre O of the square. **[Karnataka 88]**

(Ans. 6.27×10^{-7} J)

16. Positive charges of 6, 12 and 24 nC are placed at the three vertices of a square. What charge must be placed at the fourth vertex so that total potential at the centre of the square is zero ? (Ans. -42 nC)

17. Two equal charges, 2.0×10^{-7} C each are held fixed at a separation of 20 cm. A third charge of equal magnitude is placed midway between the two charges. It is now moved to a point 20 cm from both the charges. How much work is done by the electric field during the process ? (Ans. -3.6×10^{-3} J)

18. ABC is a right-angled triangle, where AB and BC are 25 cm and 60 cm respectively ; a metal sphere of 2 cm radius charged to a potential of 9×10^5 V is placed at B. Find the amount of work done in carrying a positive charge of 1 C from C to A.

(Ans. 0.042 J)

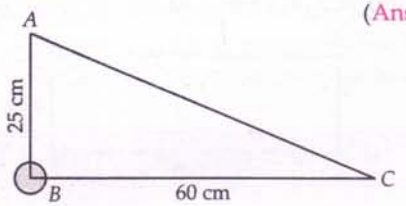


Fig. 2.18

HINTS

1. $V = \frac{W}{q} = \frac{6\text{ J}}{3\text{ C}} = \mathbf{2\ V}.$

2. As $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$ $\quad \therefore \quad 50 = 9 \times 10^9 \times \frac{q}{0.9}$

or $q = \frac{50 \times 0.9}{9 \times 10^9} = \mathbf{5 \times 10^{-9}\ C}$

As the potential is positive, the charge q must be positive.

3. Electric field of a point charge,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} = 20\ \text{NC}^{-1}$$

Electric potential of a point charge,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = 10\ \text{JC}^{-1}$$

Clearly, distance $r = \frac{V}{E} = \frac{10}{20} = \mathbf{0.5\ m}$

Magnitude of charge,

$$q = 4\pi\varepsilon_0 \cdot V.r = \frac{10 \times 0.5}{9 \times 10^9} = \mathbf{0.55 \times 10^{-9}\ C.}$$

4. Here $q = 2\,\mu\text{C} = 2 \times 10^{-6}$ C, $r_A = 2$ m, $r_B = 1$ m

$$V_A - V_B = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

$$= 2 \times 10^{-6} \times 9 \times 10^9 \left[\frac{1}{2} - \frac{1}{1}\right] \text{V}$$

$$= \mathbf{-9 \times 10^3\ V.}$$

5. (*i*) Potential at the surface,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = \frac{4 \times 10^{-7} \times 9 \times 10^9}{0.1}$$

$$= 36000\ \text{V} = \mathbf{36\ kV.}$$

(*ii*) Potential inside a hollow conductor is the same as on its surface.

(*iii*) When $r = 0.6$ m,

$$V = \frac{9 \times 10^9 \times 4 \times 10^{-7}}{0.6} = 6000\ \text{V} = \mathbf{6\ kV.}$$

6. $V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$

$$= 9 \times 10^9 \left[\frac{10 \times 10^{-6}}{0.01} + \frac{20 \times 10^{-6}}{0.01}\right]$$

$$= 27 \times 10^6\ \text{V} = \mathbf{27\ MV.}$$

7. Let the point P of zero potential lie at distance x from the charge q.

$$\therefore \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{x} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{(-2q)}{(d-x)} = 0 \text{ or } \frac{1}{x} = \frac{2}{d-x} \text{ or } x = \frac{d}{3}.$$

8. Suppose the point of zero potential is located at distance x metre from the charge of + 100 μC. Then

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{100 \times 10^{-6}}{x} - \frac{400 \times 10^{-6}}{0.30 - x}\right] = 0.$$

This gives $x = 0.06$ m = 6 cm *i.e.*, the point of zero potential lies at 6 cm from the charge of + 100 μC.

9. $V_A - V_B = 9 \times 10^9 \left[\frac{1.0 \times 10^{-6}}{2.0} - \frac{1.0 \times 10^{-6}}{1.0}\right]$

$$= -4500\ \text{V}$$

As potential is a scalar quantity, so change in position of the charge does not affect the value of potential.

10. $V_B - V_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{+q}{d} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{-q}{d} = \frac{2q}{4\pi\varepsilon_0 d}.$

11. $V_A = 9 \times 10^9 \left[\frac{2 \times 10^{-6}}{0.15} - \frac{5 \times 10^{-6}}{0.05}\right] = -7.8 \times 10^5$ V

$$V_C = 9 \times 10^9 \left[\frac{2 \times 10^{-6}}{0.05} - \frac{5 \times 10^{-6}}{0.15}\right] = 0.6 \times 10^5\ \text{V}$$

$$W = q(V_C - V_A) = 3.0 \times 10^{-6}(0.6 \times 10^5 + 7.8 \times 10^5)$$

$$= \mathbf{2.52\ J.}$$

12. $V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{AC} + \frac{q_2}{BC}\right]$

$$= 9 \times 10^9 \left[\frac{2.0 \times 10^{-6}}{\sqrt{2} \times 0.05} + \frac{1.0 \times 10^{-6}}{0.05}\right]$$

$$= 9000\left[\frac{2 + \sqrt{2}}{\sqrt{2} \times 0.05}\right] \text{V}$$

$$V_D = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{AD} + \frac{q_2}{BD}\right]$$

$$= 9 \times 10^9 \left[\frac{2.0 \times 10^{-6}}{0.05} + \frac{1.0 \times 10^{-6}}{\sqrt{2} \times 0.05}\right]$$

$$= 9000\left[\frac{2\sqrt{2} + 1}{\sqrt{2} \times 0.05}\right] \text{V}$$

$$W = q(V_D - V_C)$$

$$= 1.0 \times 10^{-6} \times 9000 \left[\frac{2\sqrt{2} + 1 - 2 - \sqrt{2}}{\sqrt{2} \times 0.05}\right]$$

$$= \mathbf{0.053\ J.}$$

13. Diagonal of the square $= \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = 2$ m

Distance of each charge from the centre,

r = Half diagonal = 1 m

∴ Potential at the centre of the square is

$$V = 9 \times 10^9 \left[\frac{2 \times 10^{-6}}{1} - \frac{2 \times 10^{-6}}{1} - \frac{3 \times 10^{-6}}{1} + \frac{6 \times 10^{-6}}{1}\right]$$

$$= \mathbf{2.7 \times 10^4\ V.}$$

14. $AC = \sqrt{4^2 + 3^2} = 5\text{ cm} = 0.05\text{ m},\ AD = BC = 0.03$ m

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{1.0 \times 10^{-11}}{0.04} - \frac{2.0 \times 10^{-11}}{0.05} + \frac{1.0 \times 10^{-11}}{0.03}\right]$$

$$= \mathbf{1.65\ V.}$$

15. $$V_D = 9 \times 10^9 \left[\frac{2 \times 10^{-9}}{0.2} + \frac{4 \times 10^{-9}}{0.2\sqrt{2}} + \frac{8 \times 10^{-9}}{0.2}\right]$$

$$= 577.26\text{ V}$$

$$V_O = 9 \times 10^9 \left[\frac{2 \times 10^{-9}}{0.1\sqrt{2}} + \frac{4 \times 10^{-9}}{0.1\sqrt{2}} + \frac{8 \times 10^{-9}}{0.1\sqrt{2}}\right]$$

$$= 890.82\text{ V}$$

$$W = q[V_O - V_D] = 2 \times 10^{-9}\,[890.82 - 577.26]$$

$$= \mathbf{6.27 \times 10^{-7}\ J.}$$

16. Suppose a charge of q nC be placed at the fourth vertex.

Let length of half diagonal be x metre.

$$V_0 = 9 \times 10^9 \left[\frac{6 \times 10^{-9}}{x} + \frac{12 \times 10^{-9}}{x} + \frac{24 \times 10^{-9}}{x} + \frac{q \times 10^{-9}}{x}\right] = 0$$

or $$\frac{6}{x} + \frac{12}{x} + \frac{24}{x} + \frac{q}{x} = 0$$

or $$\frac{q}{x} = -\frac{42}{x}$$

or $$q = \mathbf{-42\ nC.}$$

17. The situation is shown in Fig. 2.19.

$$V_C - V_D = \frac{1}{4\pi\varepsilon_0}\left[\frac{2 \times 10^{-7}}{0.20} + \frac{2 \times 10^{-7}}{0.20}\right] - \frac{1}{4\pi\varepsilon_0}\left[\frac{2 \times 10^{-7}}{0.10} + \frac{2 \times 10^{-7}}{0.10}\right]$$

$$= -1.8 \times 10^{-4}\text{ V}$$

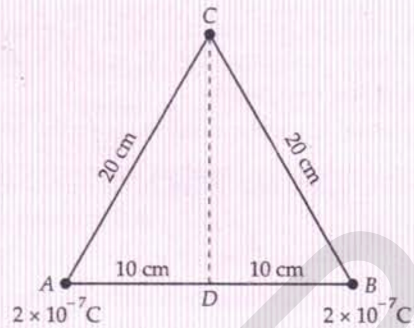


Fig. 2.19

$$W = q(V_C - V_D)$$

$$= -2 \times 10^{-7} \times 1.8 \times 10^4 = \mathbf{-3.6 \times 10^{-3}\ J.}$$

18. Potential of the charged sphere is

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

∴ $$9 \times 10^5 = 9 \times 10^9 \times \frac{q}{0.02}$$

or $$q = \frac{0.02}{10^4} = 2 \times 10^{-6} = 2\,\mu\text{C}$$

Potential at A due to charge q is

$$V_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{0.25}\text{ V}$$

Potential at C due to charge q is

$$V_C = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{0.60}\text{ V}$$

Potential difference between A and C is

$$V_A - V_C = 1.8 \times 10^{-3}\left[\frac{1}{0.25} - \frac{1}{0.60}\right]\text{V}$$

$$= \frac{1.8 \times 7}{300}\text{ V} = 0.042\text{ V}$$

Work done in moving a charge of + 1 C from C to A

$$W = q(V_A - V_C) = 1 \times 0.042 = \mathbf{0.042\ J.}$$

2.7 RELATION BETWEEN ELECTRIC FIELD AND POTENTIAL

10. *Show that the electric field at any point is equal to the negative of the potential gradient at that point.*

Computing electric field from electric potential. As shown in Fig. 2.20, consider the electric field due to charge $+q$ located at the origin O. Let A and B be two adjacent points separated by distance dr. The two

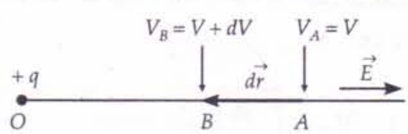


Fig. 2.20 Relation between potential and field.

points are so close that electric field $\vec{E}$ between them remains almost constant. Let V and $V + dV$ be the potentials at the two points.

The external force required to move the test charge q_0 (without acceleration) against the electric field $\vec{E}$ is given by

$$\vec{F} = -q_0 \vec{E}$$

The work done to move the test charge from A to B is

$$W = F \cdot dr = -q_0 E \cdot dr$$

Also, the work in moving the test charge from A to B is

$$W = \text{Charge} \times \text{potential difference}$$
$$= q_0 (V_B - V_A) = q_0 \, dV$$

Equating the two works done, we get

$$-q_0 E \cdot dr = q_0 \cdot dV$$

or
$$E = -\frac{dV}{dr}$$

The quantity $\frac{dV}{dr}$ is the rate of change of potential with distance and is called *potential gradient*. Thus *the electric field at any point is equal to the negative of the potential gradient at that point*. The negative sign shows that the direction of the electric field is in the direction of decreasing potential. Moreover, the field is in the direction where this decrease is steepest.

From the above relation between electric field and potential, we can draw the following important conclusions :

(*i*) Electric field is in that direction in which the potential decrease is steepest.

(*ii*) The magnitude of electric field is equal to the change in the magnitude of potential per unit displacement (called potential gradient) normal to the equipotential surface at the given point.

11. *How can we determine electric potential if electric field is known at any point ?*

Computing electric potential from electric field. The relation between electric field and potential is

$$\vec{E} = -\frac{dV}{d\vec{r}} \quad \text{or} \quad dV = -\vec{E} \cdot d\vec{r}$$

Integrating the above equation between points $\vec{r}_1$ and $\vec{r}_2$, we get

$$\int_{V_1}^{V_2} dV = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{E} \cdot d\vec{r}$$

or
$$V_2 - V_1 = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{E} \cdot d\vec{r}$$

where V_1 and V_2 are the potentials at $\vec{r}_1$ and $\vec{r}_2$ respectively. If we take $\vec{r}_1$ at infinity, then $V_1 = 0$ and put $\vec{r}_2 = \vec{r}$, we get

$$V(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{E} \cdot d\vec{r}$$

Hence by knowing electric field at any point, we can evaluate the electric potential at that point.

12. *Show that the units volt/metre and newton/coulomb are equivalent. To which physical quantity do they refer ?*

SI units of electric field. Electric field at any point is equal to the negative of the potential gradient. It suggests that the SI unit of electric field is *volt per metre*. But electric field is also defined as the force experienced by a unit positive charge, so SI unit of electric field is *newton per coulomb*. Both of these units are equivalent as shown below.

$$\frac{\text{volt}}{\text{metre}} = \frac{\text{joule / coulomb}}{\text{metre}}$$
$$= \frac{\text{newton - metre}}{\text{coulomb - metre}} = \frac{\text{newton}}{\text{coulomb}}$$

or
$$1 \text{ Vm}^{-1} = 1 \text{ NC}^{-1}$$

Examples based on
Relation between Electric Field and Potential

Formulae Used

1. Electric field in a region can be determined from the electric potential by using relation,
$$E = -\frac{dV}{dr}$$
or
$$E_x = -\frac{\partial V}{\partial x}, \; E_y = -\frac{\partial V}{\partial y}, \; E_z = -\frac{\partial V}{\partial z}$$

2. Electric field between two parallel conductors,
$$E = \frac{V}{d}$$

3. Electric potential in a region can be determined from the electric field by using the relation,
$$V = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}$$

Units Used

E is in NC^{-1} or Vm^{-1}, V in volt, r in metre.

Example 15. *Find the electric field between two metal plates 3 mm apart, connected to 12 V battery.*

Solution. Electric field,

$$E = \frac{V}{d} = \frac{12 \text{ V}}{3 \times 10^{-3} \text{ m}} = \mathbf{4 \times 10^3 \ Vm^{-1}}$$

Example 16. *Calculate the voltage needed to balance an oil drop carrying 10 electrons when located between the plates of a capacitor which are 5 mm apart ($g = 10 \ ms^{-2}$). The mass of oil drop is 3×10^{-16} kg.*

Solution. $q = ne = 10 \times 1.6 \times 10^{-19}$ C,

$$m = 3 \times 10^{-16} \text{ kg}, \ d = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$$

$$E = \frac{V}{d} = \frac{V}{5 \times 10^{-3}} \text{ Vm}^{-1}$$

For the charged oil drop to remain stationary in electric field,

$$qE = mg$$

$$\therefore \quad 10 \times 1.6 \times 10^{-19} \times \frac{V}{5 \times 10^{-3}} = 3 \times 10^{-16} \times 10$$

or $$V = \frac{3 \times 10^{-16} \times 10 \times 5 \times 10^{-3}}{10 \times 1.6 \times 10^{-19}} = \mathbf{9.47 \ V.}$$

Example 17. *An infinite plane sheet of charge density 10^{-8} Cm^{-2} is held in air. In this situation how far apart are two equipotential surfaces, whose p.d. is 5 V ?*

Solution. Electric field of an infinite plane sheet of charge,

$$E = \frac{\sigma}{2\varepsilon_0}$$

If Δr is the separation between two equipotential surfaces having potential difference ΔV, then

$$E = \frac{\Delta V}{\Delta r}$$

$$\therefore \quad \frac{\sigma}{2\varepsilon_0} = \frac{\Delta V}{\Delta r}$$

or $$\Delta r = \frac{2\varepsilon_0 \Delta V}{\sigma} = \frac{2 \times 8.85 \times 10^{-12} \times 5}{10^{-8}}$$

$$= 8.85 \times 10^{-3} \text{ m} = \mathbf{8.85 \ mm.}$$

Example 18. *A spark passes in air when the potential gradient at the surface of a charged conductor is 3×10^6 Vm^{-1}. What must be the radius of an insulated metal sphere which can be charged to a potential of 3×10^6 V before sparking into air ?*

Solution. Potential gradient,

$$\frac{dV}{dr} = 3 \times 10^6 \text{ Vm}^{-1}$$

or $$dV = 3 \times 10^6 \text{ dr}$$

or $$V = 3 \times 10^6 \text{ r}$$

But $$V = 3 \times 10^6 \text{ V}$$

$$\therefore \quad 3 \times 10^6 \ r = 3 \times 10^6$$

or $$r = \mathbf{1 \ m.}$$

Example 19. *A uniform electric field $\vec{E}$ of 300 NC^{-1} is directed along negative X-axis. A, B and C are three points in the field, having x and y coordinates (in metre), as shown in Fig. 2.21. Find the potential differences ΔV_{BA}, ΔV_{CB} and ΔV_{CA}.*

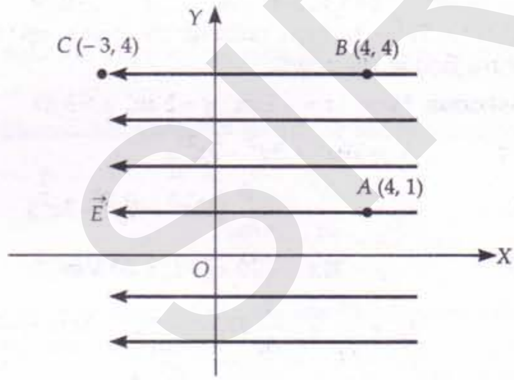


Fig. 2.21

Solution. (*i*) No work is done in moving a unit positive charge from *A* to *B* because the displacement of the charge is perpendicular to the electric field. Thus the points *A* and *B* are at the same potential.

$$\therefore \quad \Delta V_{BA} = 0$$

(*ii*) Work is done by the electric field as the positive charge moves from *B* to *C* (*i.e.*, in the direction of $\vec{E}$). Thus the point *C* is at a lower potential than the point *B*.

As $$E = -\frac{\Delta V}{\Delta x}$$

$$\therefore \quad \Delta V_{CB} = -E \, \Delta x = -300 \text{ NC}^{-1} \times 7 \text{ m}$$

$$= \mathbf{-2100 \ V.}$$

(*iii*) Points *A* and *B* lie on an equipotential surface.

So $$V_B = V_A$$

$$\Delta V_{CA} = V_C - V_A = V_C - V_B = \Delta V_{CB}$$

$$= \mathbf{-2100 \ V.}$$

Example 20. *Three points A, B and C lie in a uniform electric field (E) of 5×10^3 NC^{-1} as shown in the figure. Find the potential difference between A and C.* [CBSE F 09]

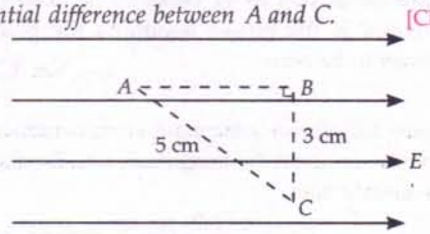


Fig. 2.22

Solution. Points B and C lie on an equipotential surface, so $V_C = V_B$.

$\therefore$ P.D. between A and C = P.D. between A and B

$$= -E\Delta x$$

$$= -5\times10^{3}\,\mathrm{NC^{-1}}\times 4\times10^{-2}\,\mathrm{m} \qquad \left[\because E = -\frac{\Delta V}{\Delta x}\right]$$

$$= \mathbf{-200\ V.} \qquad [\Delta x = AB = \sqrt{5^2-3^2} = 4\ \mathrm{cm}]$$

Example 21. *If the potential in the region of space around the point $(-1\,m, 2\,m, 3\,m)$ is given by $V = (10x^2 + 5y^2 - 3z^2)$ volt, calculate the three components of electric field at this point.*

Solution. Here $x = -1$ m, $y = 2$ m, $z = 3$ m

As $\quad V = 10x^2 + 5y^2 - 3z^2$

$$\therefore \quad E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}(10x^2 + 5y^2 - 3z^2)$$

$$= -20x = -20\times(-1) = \mathbf{20\ Vm^{-1}}.$$

$$E_y = \frac{\partial V}{\partial y} = -\frac{\partial}{dy}(10x^2 + 5y^2 - 3z^2) = -10y$$

$$= -10\times 2 = \mathbf{-20\ Vm^{-1}}$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial}{dz}(10x^2 + 5y^2 - 3z^2) = 6z$$

$$= 6\times 3 = \mathbf{18\ Vm^{-1}}.$$

Problems For Practice

1. A uniform electric field of 20 $\mathrm{NC^{-1}}$ exists in the vertically downward direction. Determine the increase in the electric potential as one goes up through a height of 50 cm. (Ans. 10 V)
2. A uniform electric field of 30 $\mathrm{NC^{-1}}$ exists along the X-axis. Calculate the potential difference $V_B - V_A$ between the points A (4 m, 2 m) and B (10 m, 5 m). (Ans. – 180 V)
3. An electric field $\vec{E} = 20\,\hat{i} + 30\,\hat{j}\ \mathrm{NC^{-1}}$ exists in free space. If the potential at the origin is taken zero, determine the potential at point (2 m, 2 m). (Ans. – 100 V)
4. The electric field in a region is given by $\vec{E} = \frac{A}{x^3}\hat{i}$. Write the SI unit for A. Write an expression for the potential in the region assuming the potential at infinity to be zero. $\left(\text{Ans. } \mathrm{Nm^3C^{-1}}, \frac{A}{2x^2}\right)$
5. Figure 2.23 shows some equipotential surfaces. What can you say about the magnitude and the direction of the electric field ? (Ans. $E = \frac{6}{r^2}\ \mathrm{Vm^{-1}}$, radially outward)

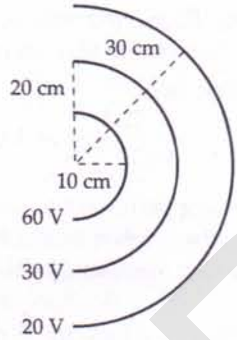


Fig. 2.23

HINTS

1. $\Delta V = -E\Delta r = -20\times\left(-\frac{50}{100}\right) = \mathbf{10\ V.}$
2. $\Delta V = -E_x\,\Delta x = -30\,(10-4) = \mathbf{-180\ V.}$
3. $\Delta V = -E_x\,\Delta x - E_y\,\Delta y = -20\times 2 - 30\times 2 = \mathbf{-100\ V.}$
4. $\because$ SI unit of electric field = $\mathrm{NC^{-1}}$
   $\therefore$ SI unit of $A = \mathrm{NC^{-1}}\times\mathrm{m^3} = \mathrm{Nm^3\,C^{-1}}$
   Potential, $V = -\int_{\infty}^{(x,y,z)} \frac{A\,dx}{x^3} = \frac{A}{2x^2}$.
5. For the equipotential surface of 60 V,
   $$60\ \mathrm{V} = \frac{kq}{r} = \frac{kq}{0.10\ \mathrm{m}}$$
   or $\quad kq = 60\ \mathrm{V}\times 0.10\ \mathrm{m} = 6\ \mathrm{Vm}$
   $$\therefore \quad E = \frac{kq}{r^2} = \frac{6}{r^2}\ \mathrm{Vm^{-1}}$$
   Clearly, E decreases with r. The direction of electric field will be radially outward because V decreases with r.

2.8 EQUIPOTENTIAL SURFACES AND THEIR PROPERTIES

13. *What is an equipotential surface ? Give an example.*

Equipotential surface. *Any surface that has same electric potential at every point on it is called an equipotential surface.* The surface may be surface of a body or a surface in space. For example, as we shall see later on, the surface of a charged conductor is an equipotential surface. By joining points of constant potential, we can draw equipotential surfaces throughout the region in which an electric field exists.

14. *State and prove the important properties of equipotential surfaces.*

Properties of equipotential surfaces : 1. ***No work is done in moving a test charge over an equipotential surface.***

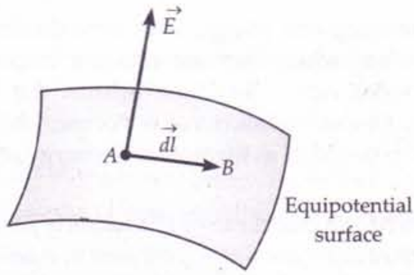


Fig. 2.24 An equipotential surface.

Let A and B be two points over an equipotential surface, as shown in Fig. 2.24. If the test charge q_0 is moved from A to B, the work done will be

$$W_{AB} = \text{Charge} \times \text{potential difference}$$
$$= q_0 (V_B - V_A)$$

As the surface is equipotential, so $V_B - V_A = 0$

Hence $W_{AB} = 0$.

2. ***Electric field is always normal to the equipotential surface at every point.*** If the field were not normal to the equipotential surface, it would have a non-zero component along the surface. So to move a test charge against this component, a work would have to be done. But there is no potential difference between any two points on an equipotential surface and consequently no work is required to move a test charge on the surface. Hence the electric field must be normal to the equipotential surface at every point.

3. ***Equipotential surfaces are closer together in the regions of strong field and farther apart in the regions of weak field.*** We know that electric field at any point is equal to the negative of potential gradient at that point.

i.e., $$E = -\frac{dV}{dr} \quad \text{or} \quad dr = -\frac{dV}{E}$$

For the same change in the value of dV *i.e.,* when dV = constant, we have

$$dr \propto \frac{1}{E}$$

Thus the spacing between the equipotential surfaces will be smaller in the regions, where the electric field is stronger and vice versa.

4. ***No two equipotential surfaces can intersect each other.*** If they interesect, then there will be two values of electric potential at the point of intersection, which is impossible.

2.9 EQUIPOTENTIAL SURFACES OF VARIOUS CHARGE SYSTEMS

15. *Sketch and explain the equipotential surfaces for : (i) a point charge, (ii) two point charges + q and − q, separated by a small distance, (iii) two point charges + q and + q separated by a small distance and (iv) a uniform electric field.*

Equipotential surfaces of various charge systems. For the various charge systems, we represent equipotential surfaces by dashed curves and lines of force by full line curves. Between any two adjacent equipotential surfaces, we assume a constant potential difference.

(*i*) ***Equipotential surfaces of a positive point charge.*** The electric potential due to a point charge q at distance r from it is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

This shows that V is constant if r is constant. Thus, the equipotential surfaces of a single point charge are concentric spherical shells with their centres at the point charge, as shown in Fig. 2.25. As the lines of force point radially outwards, so they are perpendicular to the equipotential surfaces at all points.

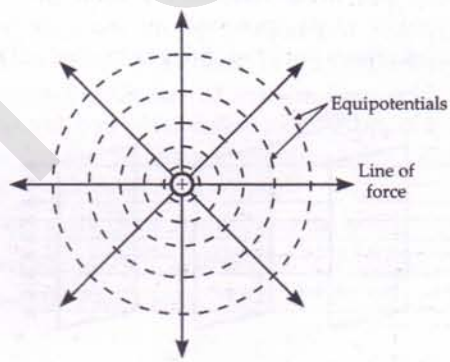


Fig. 2.25 Equipotential surface of a +ve point charge.

(*ii*) ***Equipotential surfaces of two equal and opposite point charges : Electric dipole.*** Fig. 2.26 shows the equipotential surfaces of two equal and opposite charges, $+q$ and $-q$, separated by a small distance. They are close together in the region in between the two charges.

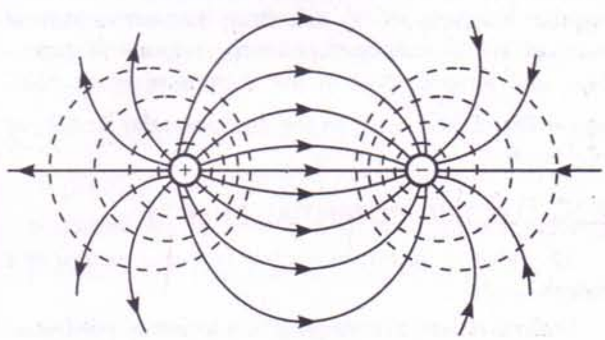


Fig. 2.26 Equipotential surfaces for two equal and opposite charges.

(*iii*) ***Equipotential surfaces of two equal positive charges.*** Fig. 2.27 shows the equipotential surfaces of two equal and positive charges, each equal to $+q$, separated by a small distance. The equipotential surfaces are far apart in the regions in between the two charges, indicating a weak field in such regions.

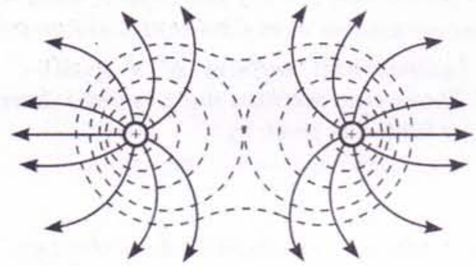


Fig. 2.27

(*iv*) ***Equipotential surfaces for a uniform electric field.*** Fig. 2.28 shows the equipotential surfaces for a uniform electric field. The lines of force are parallel straight lines and equipotential surfaces are equidistant parallel planes perpendicular to the lines of force.

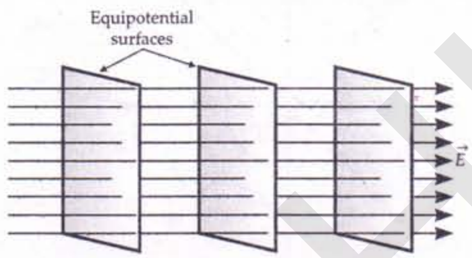


Fig. 2.28 Equipotential surfaces for a uniform electric field.

16. *Give the importance of equipotential surfaces.*

Importance of equipotential surfaces. Like the lines of force, the equipotential surfaces give a visual picture of both the direction and the magnitude of field $\vec{E}$ in a region of space. If we draw equipotential surfaces at regular intervals of V, we find that equipotential surfaces are closer together in the regions of strong field and farther apart in the regions of weak field. Moreover, $\vec{E}$ is normal to the equipotential surface at every point.

2.10 ELECTRIC POTENTIAL ENERGY

17. *What is meant by electric potential energy of a charge system ?*

Electric potential energy. It is the energy possessed by a system of charges by virtue of their positions. When two like charges lie infinite distance apart, their potential energy is zero because no work has to be done in moving one charge at infinite distance from the other. But when they are brought closer to one another, work has to be done against the force of repulsion. As electrostatic force is a conservative force, this work gets stored as the potential energy of the two charges.

*The **electric potential** energy of a system of point charges may be defined as the amount of work done in assembling the charges at their locations by bringing them in, from infinity.*

18. *Deduce expressions for the potential energy of a system of two point charges and three point charges and hence generalise the result for a system of N point charges.*

Potential energy of a system of two point charges. Suppose a point charge q_1 is at rest at a point P_1 in space, as shown in Fig. 2.29. It takes no work to bring the first charge q_1 because there is no field yet to work against.

$\therefore \quad W_1 = 0$

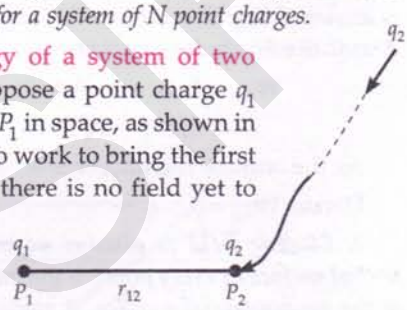


Fig. 2.29 P.E. of two point charges.

Electric potential due to charge q_1 at a point P_2 at distance r_{12} from P_1 will be

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_{12}}$$

If charge q_2 is moved in from infinity to point P_2, the work required is

$$W_2 = \text{Potential} \times \text{charge}$$

$$= V_1 \times q_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$

As the work done is stored as the potential energy U of the system $(q_1 + q_2)$, so

$$U = W_1 + W_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}.$$

Potential energy of a system of three point charges. As shown in Fig. 2.30, now we bring in the charge q_3 from infinity to the point P_3. Work has to be done against the forces exerted by q_1 and q_2.

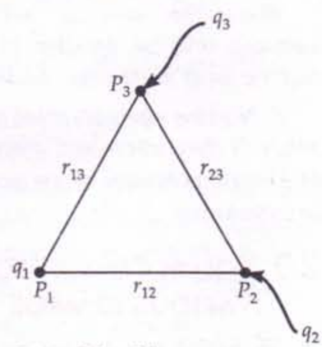


Fig. 2.30 P.E. of three point charges.

Therefore

W_3 = Potential at point P_3 due to q_1 and q_2 × charge q_3

or $$W_3 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right] \times q_3 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

Hence the electrostatic potential energy of the system $q_1 + q_2 + q_3$ is

U = Total work done to assemble the three charges

$= W_1 + W_2 + W_3$

or $$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

Potential energy of a system of N point charges. The expression for the potential energy of N point charges can be written as

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{\text{all pairs}} \frac{q_i q_j}{r_{ij}} = \frac{1}{2}\cdot\frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\sum_{\substack{j=1 \\ i \neq j}}^{N} \frac{q_i q_j}{r_{ij}}$$

As double summation counts every pair twice, to avoid this the factor 1/2 has been introduced.

NOTE The potential at jth charge due to all other charges can be written as

$$V_j = \sum_{\substack{k=1 \\ k \neq j}}^{N} \frac{q_k}{r_{jk}}$$

The expression for P.E. of N point charges can be written as

$$U = \frac{1}{2}\sum_{j=1}^{N} q_j\left[\frac{1}{4\pi\,\varepsilon_0}\sum_{\substack{k=1 \\ k \neq j}}^{N} \frac{q_k}{r_{jk}}\right] = \frac{1}{2}\sum_{j=1}^{N} q_j V_j .$$

For Your Knowledge

- Electric potential energy is a scalar quantity. While finding its value, the value of various charges must be substituted with their proper signs.
- *The potential energy of two like charges ($q_1 q_2 > 0$) is positive.* As the electrostatic force is repulsive, so a positive amount of work has to be done against this force to bring the charges from infinity to a finite separation.
- *The potential energy of two unlike charges ($q_1 q_2 < 0$) is negative.* As the electrostatic force is attractive, so a positive amount of work has to be done against this force to take the charges from the given locations to infinity. Conversely, a negative amount of work is needed to bring the charges from infinity to the present locations, so the potential energy is negative.
- As electrostatic force is a conservative force, so the potential energy of a charge configuration is independent of the manner in which the charges are assembled to the present locations. The potential energy is a characteristic of the present state of configuration, not on how this state is attained.
- Positive potential energy implies that work can be obtained by releasing the charges, while negative potential energy indicates that an external agency will have to do work to separate the charges infinite distance apart.
- Electric potential is a characteristic of an electric field, it does not matter whether a charged object is placed in that field or not. It is measured in JC^{-1} or volt. On the other hand, electric potential energy is the energy of a charged object in an external electric field. More precisely, it is the energy of the system consisting of the charged object and the external electric field (or charges producing that field). It is measured in joule.

2.11 POTENTIAL ENERGY IN AN EXTERNAL FIELD

19. *Write an expression for the potential energy of a single charge in an external field. Hence define electric potential.*

Potential energy of a single charge. We wish to determine the potential energy of a charge q in an external electric field $\vec{E}$ at a point P where the corresponding external potential is V. By definition, V at a point P is the amount of work done in bringing a unit positive charge from infinity to the point P. Thus, the work done in bringing a charge q from infinity to the point P will be qV, *i.e.*, $W = qV$

This work done is stored as the potential energy of the charge q. If $\vec{r}$ is the position vector of point P relative to some origin, then

$$U(\vec{r}) = qV(\vec{r})$$

P.E. of a charge in an external field

= Charge × external electric potential

As $$V = \frac{U}{q}$$

So we can define ***electric potential*** *at a given point in an external field as the potential energy of a unit positive charge at that point.*

20. *Write an expression for the potential energy of two point charges q_1 and q_2, separated by distance r in an electric field $\vec{E}$.*

Potential energy of a system of two point charges in an external field. Let $V(\vec{r_1})$ and $V(\vec{r_2})$ be the electric potentials of the field $\vec{E}$ at the points having position vectors $\vec{r_1}$ and $\vec{r_2}$ as shown in Fig. 2.31.

Work done in bringing q_1 from ∞ to $\vec{r_1}$ against the external field

$$= q_1 V(\vec{r_1})$$

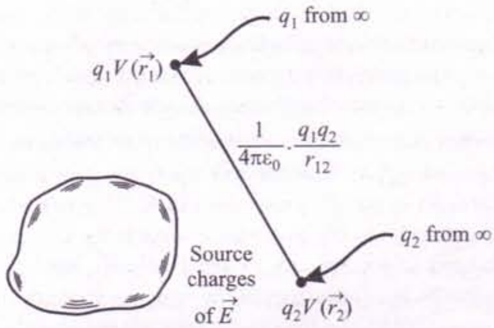


Fig. 2.31 P.E. of two charges in an external field.

Work done in bringing q_2 from ∞ to $\vec{r}_2$ against the external field

$$= q_2 V(\vec{r}_2)$$

Work done on q_2 against the force exerted by q_1

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$

where r_{12} is the distance between q_1 and q_2.

Total potential energy of the system = The work done in assembling the two charges

or $$U = q_1 V(\vec{r}_1) + q_2 V(\vec{r}_2) + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$

21. *Define electron volt. Express it in joule.*

Units of electrostatic potential energy. Suppose an electron ($q = 1.6 \times 10^{-19}$ C) is moved through a potential difference of 1 volt, then the change in its P.E. would be

$$\Delta U = q\,\Delta V = 1.6 \times 10^{-19}\ \text{C} \times 1\ \text{V} = 1.6 \times 10^{-19}\ \text{J}$$

This is a commonly used unit of energy in atomic physics and we call it ***electron volt*** (eV).

Thus electron volt is the potential energy gained or lost by an electron in moving through a potential difference of 1 volt.

$$1\ \text{eV} = 1.6 \times 10^{-19}\ \text{J}$$

Multiples and submultiples of eV

1 meV (milli electron volt)
$= 10^{-3}$ eV $= 1.6 \times 10^{-22}$ J

1 keV (kilo electron volt)
$= 10^{3}$ eV $= 1.6 \times 10^{-16}$ J

1 MeV (million electron volt)
$= 10^{6}$ eV $= 1.6 \times 10^{-13}$ J

1 GeV (giga electron volt)
$= 10^{9}$ eV $= 1.6 \times 10^{-10}$ J

1 TeV (tera electron volt)
$= 10^{12}$ eV $= 1.6 \times 10^{-7}$ J.

2.12 POTENTIAL ENERGY OF A DIPOLE IN A UNIFORM ELECTRIC FIELD

22. *Derive an expression for the potential energy of a dipole in a uniform electric field. Discuss the conditions of stable and unstable equilibrium.*

Potential energy of a dipole placed in a uniform electric field. As shown in Fig. 2.32, consider an electric dipole placed in a uniform electric field $\vec{E}$ with its dipole moment $\vec{p}$ making an angle θ with the field. Two equal and opposite forces $+q\vec{E}$ and $-q\vec{E}$ act on its two ends. The two forces form a couple. The torque exerted by the couple will be

$$\tau = qE \times 2a \sin\theta = pE \sin\theta$$

where $q \times 2a = p$, is the dipole moment.

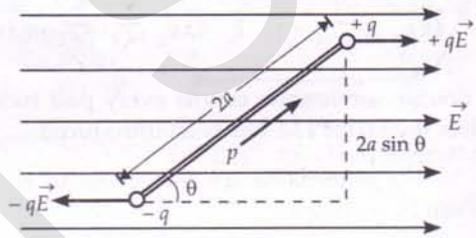


Fig. 2.32 Torque on a dipole in a uniform electric field.

If the dipole is rotated through a small angle $d\theta$ against the torque acting on it, then the small work done is

$$dW = \tau\, d\theta = pE \sin\theta\, d\theta$$

The total work done in rotating the dipole from its orientation making an angle θ_1, with the direction of the field to θ_2 will be

$$W = \int dW = \int_{\theta_1}^{\theta_2} pE \sin\theta\, d\theta$$

$$= pE\left[-\cos\theta\right]_{\theta_1}^{\theta_2} = pE(\cos\theta_1 - \cos\theta_2)$$

This work done is stored as the potential energy U of the dipole.

$$\therefore \quad U = pE(\cos\theta_1 - \cos\theta_2)$$

If initially the dipole is oriented perpendicular to the direction of the field ($\theta_1 = 90°$) and then brought to some orientation making an angle θ with the field ($\theta_2 = \theta$), then potential energy of the dipole will be

$$U = pE(\cos 90° - \cos\theta) = pE(0 - \cos\theta)$$

or $$U = -pE\cos\theta = -\vec{p} \cdot \vec{E}$$

Special Cases

1. *Position of stable equilibrium.* When $\theta = 0°$,
$$U = -pE\cos 0° = -pE$$
Thus the *potential energy of a dipole is minimum when its dipole moment is parallel to the external field*. This is the position of stable equilibrium.

2. *Position of zero energy.* When $\theta = 90°$,
$$U = -pE\cos 90° = 0.$$
Thus the potential energy of a dipole is zero when it is held perpendicular to the external field. This can be explained as follows. If we hold the dipole perpendicular to the electric field and bring it from infinity into the field, then the work done on charge $+q$ by the external agent is equal to the work done on charge $-q$. The net work done on the dipole will be zero and hence its potential energy is zero.

3. *Position of unstable equilibrium.* When $\theta = 180°$,
$$U = -pE\cos 180° = +pE$$
Thus *the potential energy of a dipole is maximum when its dipole moment is antiparallel to the external field*. This is the position of unstable equilibrium.

Examples based on
Electric Potential Energy

Formulae Used

1. Electric potential energy of a system of two point charges,
$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_{12}}$$

2. Electric potential energy of a system of N point charges,
$$U = \frac{1}{4\pi\varepsilon_0}\sum_{\text{all pairs}}\frac{q_j q_k}{r_{jk}}$$

3. Potential energy of an electric dipole in a uniform electric field,
$$U = -pE(\cos\theta_2 - \cos\theta_1)$$
If initially the dipole is perpendicular to the field E, $\theta_1 = 90°$ and $\theta_2 = \theta$ (say), then
$$U = -pE\cos\theta = -\vec{p}\cdot\vec{E}$$
If initially the dipole is parallel to the field E, $\theta_1 = 0°$ and $\theta_2 = \theta$ (say), then
$$U = -pE(\cos\theta - 1) = pE(1 - \cos\theta)$$

Units Used

Charges are in coulomb, distances in metre, energy in joule or in electron volt (eV) and dipole moment in coulomb metre (Cm).

$1\,\text{eV} = 1.6 \times 10^{-19}$ C, $1\,\text{MeV} = 1.6 \times 10^{-13}$ C.

Example 22

(*a*) *Determine the electrostatic potential energy of a system consisting of two charges 7 μC and −2 μC (and with no external field) placed at (−9 cm, 0, 0) and (9 cm, 0, 0) respectively.*

(*b*) *How much work is required to separate the two charges infinitely away from each other ?*

(*c*) *Suppose the same system of charges is now placed in an external electric field $E = A(1/r^2)$; $A = 9 \times 10^5\ Cm^{-2}$. What would the electrostatic energy of the configuration be ?* [NCERT]

Solution. (*a*) $q_1 = 7\,\mu C = 7 \times 10^{-6}$ C, $q_2 = -2 \times 10^{-6}$ C, $r = 18$ cm $= 0.18$ m

Electrostatic potential energy of the two charges is
$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r}$$
$$= \frac{9 \times 10^9 \times 7 \times 10^{-6} \times (-2) \times 10^{-6}}{0.18} = \mathbf{-0.7\ J.}$$

(*b*) Work required to separate two charges infinitely away from each other,
$$W = U_2 - U_1 = 0 - U = -(-0.7) = \mathbf{0.7\ J.}$$

(*c*) Energy of the two charges in the external electric field
= Energy of interaction of two charges with the external electric field
+ Mutual interaction energy of the two charges
$$= q_1 V(r_1) + q_2 V(r_2) + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$
$$= q_1\frac{A}{r_1} + q_2\frac{A}{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \qquad \left[V = Er = \frac{A}{r}\right]$$
$$= \left[\frac{7\mu C}{0.09\ \text{m}} + \frac{-2\mu C}{0.09\ \text{m}}\right] \times 9 \times 10^5\ \text{Cm}^{-2} - 0.7\ \text{J}$$
$$= (70 - 20) - 0.7 = 50 - 0.7 = \mathbf{49.3\ J.}$$

Example 23. *Three charges $-q$, $+Q$ and $-q$ are placed at equal distances on a straight line. If the potential energy of the system of three charges is zero, find the ratio Q/q.*

Solution. As shown in Fig. 2.33, suppose the three charges are placed at points A, B and C respectively on a straight line, such that $AB = BC = r$.

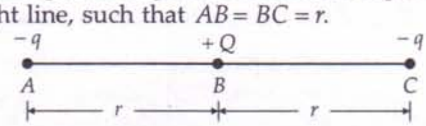


Fig. 2.33

As the total P.E. of the system is zero, so
$$\frac{1}{4\pi\varepsilon_0}\left[\frac{-qQ}{r} + \frac{(-q)(-q)}{2r} + \frac{Q(-q)}{r}\right] = 0$$
or $-Q + \dfrac{q}{2} - Q = 0$ or $2Q = \dfrac{q}{2}$ or $\dfrac{Q}{q} = \dfrac{1}{4} = \mathbf{1:4.}$

Example 24. *Two positive point charges of 0.2 μC and 0.01 μC are placed 10 cm apart. Calculate the work done in reducing the distance to 5 cm.*

Solution. Here $q_1 = 0.2 \times 10^{-6}$C, $q_2 = 0.01 \times 10^{-6}$C

Initial separation $(r_i) = 10$ cm $= 0.10$ m

Final separation $(r_f) = 5$ cm $= 0.05$ m

Work done = Change in potential energy

= Final P.E. - Initial P.E.

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_f} - \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_i} = \frac{q_1 q_2}{4\pi\varepsilon_0}\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

$$= 0.2 \times 10^{-6} \times 0.01 \times 10^{-6} \times 9 \times 10^9 \left[\frac{1}{0.05} - \frac{1}{0.10}\right]$$

$$= \mathbf{1.8 \times 10^{-4}\ J.}$$

Example 25. *Two electrons, each moving with a velocity of $10^6\ ms^{-1}$, are released towards each other. What will be the closest distance of approach between them ?*

Solution. Let r_0 be the distance of closest approach of the two electrons. At this distance, the entire K.E. of the electrons changes into their P.E. Therefore,

$$\frac{1}{2} mv^2 + \frac{1}{2} mv^2 = \frac{1}{4\pi\varepsilon_0} \frac{e\,e}{r_0}$$

$$r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{mv^2} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{9.1 \times 10^{-31} \times (10^6)^2}$$

$$= \mathbf{2.53 \times 10^{-10}\ m.}$$

Example 26. *Two particles have equal masses of 5.0 g each and opposite charges of $+4 \times 10^{-5}$C and -4.0×10^{-5}C. They are released from rest with a separation of 1.0 m between them. Find the speeds of the particles when the separation is reduced to 50 cm.*

Solution. Here $m = 5.0$ g $= 5 \times 10^{-3}$ kg,

$q = \pm 4 \times 10^{-5}$C, $r_1 = 1.0$ m, $r_2 = 50$ cm $= 0.50$ m

Let v = speed of each particle at the separation of 50 cm.

From energy conservation principle,

K.E. of the two particles at 50 cm separation

+ P.E. of the two particles at 50 cm separation

= P.E. of the two particles at 1.0 m separation

$$\frac{1}{2} mv^2 + \frac{1}{2} mv^2 + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_1}$$

$$mv^2 = \frac{q_1 q_2}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \quad \text{or} \quad v^2 = \frac{q_1 q_2}{4\pi\varepsilon_0\, m}\left[\frac{r_2 - r_1}{r_1 r_2}\right]$$

$$\therefore\ v^2 = \frac{4 \times 10^{-5} \times (-4 \times 10^{-5}) \times 9 \times 10^9}{5 \times 10^{-3}}\left[\frac{0.50 - 1.0}{1.0 \times 0.50}\right]$$

$$= 2880 \quad \text{or} \quad v = \mathbf{53.67\ ms^{-1}}.$$

Example 27. *Four charges are arranged at the corners of a square ABCD of side d as shown in Fig. 2.34. (i) Find the work required to put together this arrangement. (ii) A charge q_0 is brought to the centre E of the square, the four charges being held fixed at its corners. How much extra work is needed to do this ?* [NCERT ; CBSE F 15]

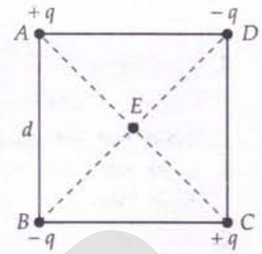


Fig. 2.34

Solution. (*i*) Given $AB = BC = CD = AD = d$

$$\therefore \quad AC = BD = \sqrt{d^2 + d^2} = \sqrt{2}\, d$$

Work required to put the four charges together

= Total electrostatic P.E. of the four charges

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A q_B}{AB} + \frac{q_A q_C}{AC} + \frac{q_A q_D}{AD} + \frac{q_B q_C}{BC} + \frac{q_B q_D}{BD} + \frac{q_C q_D}{CD}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} + \frac{q^2}{\sqrt{2}d} - \frac{q^2}{d} - \frac{q^2}{d} + \frac{q^2}{\sqrt{2}\,d} - \frac{q^2}{d}\right]$$

$$= -\frac{q^2}{4\pi\varepsilon_0}(4 - \sqrt{2}).$$

(*ii*) Extra work needed to bring charge q_0 to centre E

$W = q_0 \times$ Electrostatic potential at E due to the four charges

$$= q_0\left[\frac{q}{4\pi\varepsilon_0 (d/\sqrt{2})} + \frac{-q}{4\pi\varepsilon_0 (d/\sqrt{2})} + \frac{q}{4\pi\varepsilon_0 (d/\sqrt{2})} + \frac{-q}{4\pi\varepsilon_0 (d/\sqrt{2})}\right] = \mathbf{0}.$$

Example 28. *Three point charges, $+Q$, $+2Q$ and $-3Q$ are placed at the vertices of an equilateral triangle ABC of side l (Fig. 2.35). If these charges are displaced to the midpoints A_1, B_1 and C_1 respectively, find the amount of the work done in shifting the charges to the new locations.* [CBSE OD 2015]

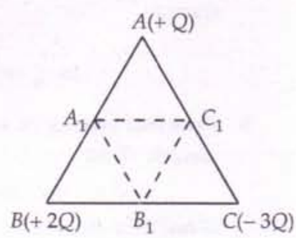


Fig. 2.34

Solution. $A_1B_1 = B_1C_1 = A_1C_1 = \frac{AB}{2} = \frac{l}{2}$

Initial P.E. of the system is

$$U_i = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q \times 2Q}{l} + \frac{2Q \times (-3Q)}{l} + \frac{Q \times (-3Q)}{l}\right]$$

$$= -\frac{1}{4\pi\varepsilon_0} \cdot \frac{7Q^2}{l}$$

Final P.E. of the system is

$$U_f = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q\times 2Q}{l/2} + \frac{2Q\times(-3Q)}{l/2} + \frac{Q\times(-3Q)}{l/2}\right]$$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot\frac{14Q^2}{l}$$

Work done $= U_f - U_i$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot\frac{14Q^2}{l} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{7Q^2}{l} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{7Q^2}{l}.$$

Example 29. *An electric dipole of length 2 cm is placed with its axis making an angle of 60° to a uniform electric field of $10^5\ NC^{-1}$. If it experiences a torque of $8\sqrt{3}$ Nm, calculate the*

(*i*) *magnitude of the charge on the dipole, and*

(*ii*) *potential energy of the dipole.* [CBSE OD 2000]

Solution. Here $2a = 2\text{ cm} = 0.02\text{ m}$, $\theta = 60°$, $E = 10^5\ \text{NC}^{-1}$, $\tau = 8\sqrt{3}$ Nm

(*i*) $\tau = pE\sin\theta = q\times 2a\times E\sin\theta$

$\therefore\quad 8\sqrt{3} = q\times 0.02\times 10^5\times\sin 60°$

or $\quad q = \dfrac{8\sqrt{3}\times 2}{0.02\times 10^5\times\sqrt{3}} = \mathbf{8\times 10^{-3}\ C.}$

(*ii*) P.E. of the dipole is

$$U = -pE\cos\theta = -q\times 2a\times E\cos\theta$$
$$= -8\times 10^{-3}\times 0.02\times 10^5\times\cos 60° = \mathbf{-8\ J.}$$

Example 30. *An electric dipole of length 4 cm, when placed with its axis making an angle of 60° with a uniform electric field experiences a torque of $4\sqrt{3}$ Nm. Calculate the (i) magnitude of the electric field, (ii) potential energy of the dipole, if the dipole has charges of ± 8 nC.* [CBSE OD 04 ; D 06C, 14]

Solution. Here $2a = 4\text{ cm} = 0.04\text{ m}$, $\theta = 60°$, $\tau = 4\sqrt{3}$ Nm, $q = 8\text{ nC} = 8\times 10^{-9}$ C

Dipole moment,

$$p = q\times 2a = 8\times 10^{-9}\times 0.04 = 0.32\times 10^{-9}\ \text{Cm}.$$

(*i*) As $\tau = pE\sin\theta$

$$\therefore\quad E = \frac{\tau}{p\sin\theta} = \frac{4\sqrt{3}}{0.32\times 10^{-9}\times\sin 60°}$$
$$= \frac{4\sqrt{3}\times 10^9\times 2}{0.32\times\sqrt{3}} = \mathbf{2.5\times 10^{10}\ NC^{-1}}.$$

(*ii*) $U = -pE\cos\theta$

$$= -0.32\times 10^{-9}\times 2.5\times 10^{10}\times\cos 60° = \mathbf{-4\ J.}$$

Example 31. *A molecule of a substance has permanent electric dipole moment equal to 10^{-29} Cm. A mole of this substance is polarized (at low temperature) by applying a strong electrostatic field of magnitude ($10^6\ Vm^{-1}$). The direction of the field is suddenly changed by an angle of 60°. Estimate the heat released by the substance in aligning its dipoles along the new direction of the field. For simplicity assume 100% polarization of the sample.* [NCERT]

Solution. Here $p = 10^{-29}$ Cm, $E = 10^6\ \text{Vm}^{-1}$, $\theta = 60°$, $N = 6\times 10^{23}$

Work required to bring one dipole from position $\theta = 0°$ to position θ is

$$W = pE - pE\cos\theta = pE(1-\cos\theta)$$
$$= 10^{-29}\times 10^6(1-\cos 60°)\ \text{J} = 0.5\times 10^{-23}\ \text{J}$$

Work required for one mole of dipoles

$$= W\times N = 0.5\times 10^{-23}\times 6\times 10^{23} = 3.0\ \text{J}$$

Heat released = Loss in P.E. = Work done = **3.0 J.**

Problems For Practice

1. Two point charges $+10\ \mu C$ and $-10\ \mu C$ are separated by a distance of 2.0 cm in air. (*i*) Calculate the potential energy of the system, assuming the zero of the potential energy to be at infinity. (*ii*) Draw an equipotential surface of the system. **[CBSE D 04]** (Ans. – 45 J)
2. Two point charges *A* and *B* of values $+15\ \mu C$ and $+9\ \mu C$ are kept 18 cm apart in air. Calculate the work done when charge *B* is moved by 3 cm towards *A*. **[CBSE OD 2000]** (Ans. 1.35 J)
3. Two point charges 20×10^{-6}C and -4×10^{-6}C are separated by a distance of 50 cm in air. (*i*) Find the point on the line joining the charges, where the electric potential is zero. (*ii*) Also find the electrostatic potential energy of the system. **[CBSE OD 08]** [Ans. (*i*) 41 cm from the charge of 20×10^{-6} C (*ii*) – 1.44 J]
4. Two charges, of magnitude 5 nC and – 2 nC, are placed at points (2 cm, 0, 0) and (*x* cm, 0, 0) in a region of space, where there is no other external field. If the electrostatic potential energy of the system is – 0.5 μJ, what is the value of *x* ? **[CBSE D 08C]** (Ans. x = 4 cm)
5. Three point charges are arranged as shown in Fig. 2.36. What is their mutual potential energy ? Take $q = 1.0\times 10^{-4}$ C and a = 10 cm. (Ans. 0.27 J)

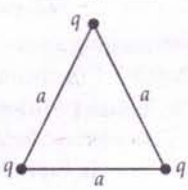


Fig. 2.36

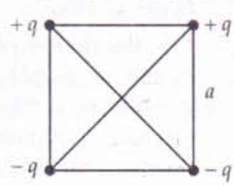
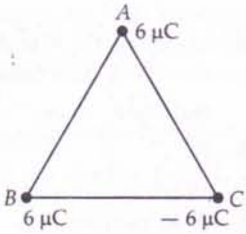


Fig. 2.37

6. Determine potential energy of the charge configuration shown in Fig. 2.37. $\left(\text{Ans. } \dfrac{q^2}{4\pi\varepsilon_0 a}(-\sqrt{2})\right)$

7. Find the amount of work done in arranging the three point charges, on the vertices of an equilateral triangle ABC, of side 10 cm, as shown in the adjacent figure. **[CBSE Sample Paper 2011]**



(Ans. – 3.24 J)

8. Calculate the work done to dissociate the system of three charges placed on the vertices of a triangle as shown in Fig. 2.38. Here $q = 1.6 \times 10^{-10}$ C. **[CBSE D 08 ; OD 13]** (Ans. 2.304×10^{-8} J)

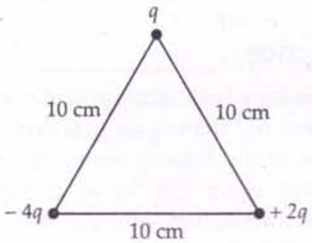


Fig. 2.38

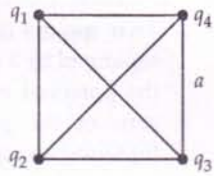


Fig. 2.39

9. What is the electrostatic potential energy of the charge configuration shown in Fig. 2.39 ? Take

$q_1 = +\,1.0 \times 10^{-8}$ C, $q_2 = -\,2.0 \times 10^{-8}$ C,

$q_3 = +\,3.0 \times 10^{-8}$ C, $q_4 = +\,2.0 \times 10^{-8}$ C

and $a = 1.0$ metre. (Ans. $-\,6.36 \times 10^{-7}$ J)

10. Three point charges $+q$, $+2q$ and Q are placed at the three vertices of an equilateral triangle. Find the value of charge Q (in terms of q), so that electric potential energy of the system is zero. (Ans. $Q = -2q/3$)

11. An electron (charge $= -e$) is placed at each of the eight corners of a cube of side a and an α-particle (charge $= +2e$) at the centre of the cube. Calculate the potential energy of the system.

(Ans. $3.89 \times 10^{10}\, e^2/a$ joule)

12. Two identical particles, each having a charge of 2.0×10^{-4} C and mass of 10 g, are kept at a separation of 10 cm and then released. What would be the speeds of the particles when the separation becomes large ? (Ans. 600 ms^{-1})

13. Find the amount of work done in rotating an electric dipole, of dipole moment 3.2×10^{-8} Cm, from its position of stable equilibrium, to the position of unstable equilibrium, in a uniform electric field of intensity 10^4 N/C. **[CBSE Sample Paper 2011]**

(Ans. 6.4×10^{-4} J)

14. An electric dipole consists of two opposite charges each of magnitude 1 μC separated by 2 cm. The dipole is placed in an external electric field of $10^5\, NC^{-1}$. Find (*i*) the maximum torque exerted by the field on the dipole (*ii*) the work which the external agent will have to do in turning the dipole through 180° starting from the position θ = 0°.

[Ans. (*i*) 2×10^{-3} Nm (*ii*) 4×10^{-3} J]

HINTS

1. $$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$$

$$= 9 \times 10^9 \times \frac{10 \times 10^{-6} \times (-10) \times 10^{-6}}{2.0 \times 10^{-2}} = -\,\mathbf{45\ J.}$$

For equipotential surface, see Fig. 2.26 on page 2.15.

2. W = Final P.E. – Initial P.E.

$$= \frac{q_1 q_2}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$= 9 \times 10^9 \times 15 \times 10^{-6} \times 9 \times 10^{-6}\left[\frac{100}{15} - \frac{100}{18}\right]$$

$= \mathbf{1.35\ J.}$

3. (*i*) Suppose the point of zero potential is located at distance x metre from the charge of 20×10^{-6} C.

Then, $$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{20 \times 10^{-6}}{x} - \frac{4 \times 10^{-6}}{0.50 - x}\right] = 0$$

This gives $x = 0.41$ m = **41 cm.**

(*ii*) $$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$$

$$= \frac{9 \times 10^9 \times 20 \times 10^{-6} \times (-4) \times 10^{-6}}{0.50} = \mathbf{-1.44\ J.}$$

4. $$U = \frac{1}{4\pi\varepsilon_0}\,\frac{q_1 q_2}{r}$$

$$\therefore\ -0.5 \times 10^{-6} = \frac{9 \times 10^9 \times 5 \times 10^{-9} \times (-2) \times 10^{-9}}{(x - 2) \times 10^{-2}}$$

On solving, $x = \mathbf{4\ cm}$

5. $$U = \frac{3}{4\pi\varepsilon_0}\,\frac{q^2}{a}$$

$$= \frac{3 \times 9 \times 10^9 \times (1.0 \times 10^{-4})^3}{0.10} = \mathbf{0.27\ J.}$$

7. $$W = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A q_B}{AB} + \frac{q_A q_C}{AC} + \frac{q_B q_C}{BC}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q \cdot q}{r} + \frac{q(-q)}{r} + \frac{q(-q)}{r}\right]$$

$$= -\frac{1}{4\pi\varepsilon_0}\,\frac{q^2}{r} = -\frac{9 \times 10^9 \times (6 \times 10^{-6})^2}{0.10}\ \text{J} = -\,\mathbf{3.24\ J.}$$

8. Initial P.E. of the three charges,

$$U_i = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r} + \frac{q_2 q_3}{r} + \frac{q_1 q_3}{r}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q(-4q)}{r} + \frac{(-4q)\times 2q}{r} + \frac{q\times 2q}{r}\right]$$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot\frac{10q^2}{r} = -\frac{9\times 10^9 \times 10 \times (1.6\times 10^{-10})^2}{0.10}\ \text{J}$$

$$= -2.304 \times 10^{-8}\ \text{J}$$

Final P.E., $U_f = 0$

Work required to dissociate the system of three charges,

$$W = U_f - U_i = \mathbf{2.304 \times 10^{-8}\ J.}$$

9. $U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{a} + \frac{q_1q_3}{\sqrt{2}a} + \frac{q_1q_4}{a} + \frac{q_2q_3}{a} + \frac{q_2q_4}{\sqrt{2}a} + \frac{q_3q_4}{a}\right]$

$$= \frac{9\times 10^9}{1.0}\left[(1)(-2) + \frac{(1)(3)}{\sqrt{2}} + (1)(2) + (-2)(3) + \frac{(-2)\times(2)}{\sqrt{2}} + (3)(2)\right]\times 10^{-16}\ \text{J}$$

$$= -\frac{9\times 10^9 \times 10^{-16}}{\sqrt{2}}\ \text{J} = \mathbf{-6.36 \times 10^{-7}\ J.}$$

10. Suppose the charges $+q$, $+2q$ and Q are placed at the corners A, B and C of an equilateral ΔABC of side a. Then

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{q\times 2q}{r} + \frac{q\times Q}{r} + \frac{2q\times Q}{r}\right] = 0$$

or $\quad 2q + Q + 2Q = 0 \quad$ or $\quad Q = \mathbf{-2q/3}$.

11. $U = 9\times 10^9\left[\frac{12(-e)(-e)}{a} + 12\frac{(-e)(-e)}{\sqrt{2}a} + 4\frac{(-e)(-e)}{\sqrt{3}a} + 8\frac{(-e)(2e)}{\sqrt{3}a/2}\right]$

$$= \frac{9\times 10^9 \times 4 \times e^2}{a}\left[3 + \frac{3}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{8}{\sqrt{3}}\right]$$

$$= \frac{36\times 10^9\, e^2}{a}[3 + 2.12 - 4.04] = 3.89 \times 10^{10}\,\frac{e^2}{a}\ \text{joule.}$$

12. Here $q = 2.0 \times 10^{-4}$ C, $m = 10\ \text{g} = 10^{-2}$ kg,
$r = 10\ \text{cm} = 0.10$ m

Let v be the speed of each particle at infinite separation. By conservation of energy,

P.E. of two particles at the separation of 10 cm
= K.E. of the two particles at infinite separation

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

or $\quad v^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{rm}$

$$= \frac{9\times 10^9 \times 2.0 \times 10^{-4} \times 2.0 \times 10^{-4}}{0.10 \times 10^{-2}} = 36 \times 10^4$$

$\therefore \quad v = \mathbf{600\ ms^{-1}}$.

13. Here $\theta_1 = 0°$, $\theta_2 = 180°$, $p = 3.2 \times 10^{-8}$ Cm, $E = 10^4$ N/C

$$W = pE(\cos\theta_1 - \cos\theta_2)$$
$$= 3.2\times 10^{-8} \times 10^4(\cos 0° - \cos 180°)$$
$$= 3.2 \times 10^{-4} \times (1+1) = \mathbf{6.4 \times 10^{-4}\ J.}$$

14. $p = q \times 2a = 10^{-6} \times 0.02 = 2 \times 10^{-8}$ Cm

(i) $\tau_{max} = pE\sin 90° = 2\times 10^{-8} \times 10^5 \times 1 = \mathbf{2 \times 10^{-3}\ Nm.}$

(ii) $W = pE(\cos\theta_1 - \cos\theta_2)$
$= 2\times 10^{-8} \times 10^5(\cos 0° - \cos 180°)$
$= 2\times 10^{-3}(1+1) = \mathbf{4 \times 10^{-3}\ J.}$

2.13 CONDUCTORS AND INSULATORS

23. *What are conductors and insulators ? Why were insulators called dielectrics and conductors non-electrics ?*

Conductors and insulators. On the basis of their behaviour in an external electric field, most of the materials can be broadly classified into *two* categories :

1. ***Conductors.*** *These are the substances which allow large scale physical movement of electric charges through them when an external electric field is applied.* For example, silver, copper, aluminium, graphite, human body, acids, alkalies, etc.

2. ***Insulators.*** *These are the substances which do not allow physical movement of electric charges through them when an external electric field is applied.* For example, diamond, glass, wood, mica, wax, distilled water, ebonite, etc.

The rubbed insulators were able to retain charges placed on them, so they were called *dielectrics*. The rubbed conductors (metals) could not retain charges placed on them but immediately drained away the charges, so they were called *non-electrics*.

2.14 FREE AND BOUND CHARGES

24. *Discuss the various free and bound charges present in conductors and insulators.*

Free and bound charges. The difference between the electrical behaviour of conductors and insulators can be understood on the basis of free and bound charges.

In ***metallic conductors***, the electrons of the outer shells of the atoms are loosely bound to the nucleus. They get detached from the atoms and move almost freely inside the metal. In an external electric field, these free electrons drift in the opposite direction of the electric field. The *positive ions* which consist of nuclei and electrons of inner shells remain held in their fixed positions. These immobile charges constitute the *bound charges*.

In ***electrolytic conductors,*** both positive and negative ions act as charge carriers. However, their movements are restricted by the external electric field and the electrostatic forces between them.

In *insulators,* the electrons are tightly bound to the nuclei and cannot be detached from the atoms, *i.e.,* charges in insulators are bound charges. Due to the absence of free charges, insulators are poor conductors of electricity.

For Your Knowledge

- A third important category of materials is the semiconductors which we shall discuss in chapter 14.
- In metallic conductors, electrons of outer shells of the atoms are the free charges while the immobile positive ions are the bound charges.
- In electrolytic conductors, both positive and negative ions are the free charges.
- In insulators, both electrons and the positive ions are the bound charges.
- There is no clear cut distinction between conductors and insulators – their electrical properties vary continuously within a very large range. For example, the ratio of the electrical properties between a metal and glass may be as high as 10^{20}.

2.15 BEHAVIOUR OF CONDUCTORS IN ELECTROSTATIC FIELDS

25. *State and prove the various electrostatic properties shown by conductors placed in electrostatic fields.*

Electrostatic properties of a conductor. When placed in electrostatic fields, the conductors show the following properties :

1. ***Net electrostatic field is zero in the interior of a conductor.*** As shown in Fig. 2.40, when a conductor is placed in an electric field $\vec{E}_{ext}$, its free electrons begin to move in the opposite direction of $\vec{E}_{ext}$. Negative charges are induced on the left end and positive charges are induced on the right end of the conductor. The process continues till the electric field $\vec{E}_{ind}$ set up by the induced charges becomes equal and opposite to the field $\vec{E}_{ext}$. The net field $\vec{E}\ (= \vec{E}_{ext} - \vec{E}_{ind})$ inside the conductor will be zero.

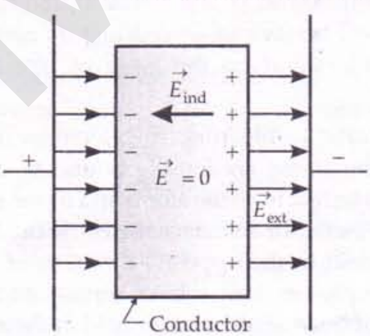


Fig. 2.40 Electric field inside a conductor is zero.

2. ***Just outside the surface of a charged conductor, electric field is normal to the surface.*** If the electric field is not normal to the surface, it will have a component tangential to the surface which will immediately cause the flow of charges, producing surface currents. But no such currents can exist under static conditions. Hence electric field is normal to the surface of the conductor at every point.

3. ***The net charge in the interior of a conductor is zero and any excess charge resides at its surface.*** As shown in Fig. 2.41, consider a conductor carrying an excess charge q with no currents flowing in it. Choose a Gaussian surface inside the conductor just near its outer boundary. As the field $\vec{E} = 0$ at all points inside the conductor, the flux ϕ_E through the Gaussian surface must be zero. According to Gauss's theorem,

$$\phi_E = \oint \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

As $\quad \phi_E = 0, \quad$ so $\quad q = 0$

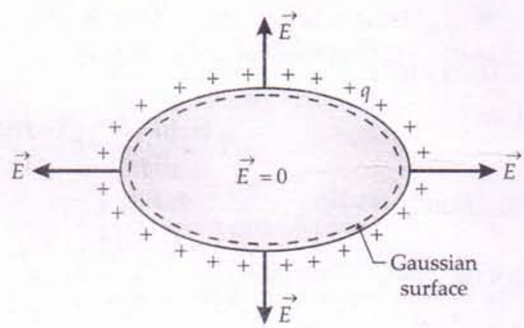


Fig. 2.41

Hence there can be no charge in the interior of the conductor because the Gaussian surface lies just near the outer boundary. The entire excess charge q must reside at the surface of the conductor.

4. ***Potential is constant within and on the surface of a conductor.*** Electric field at any point is equal to the negative of the potential gradient,

i.e., $$E = -\frac{dV}{dr}$$

But inside a conductor $E = 0$ and moreover, E has no tangential component on the surface, so

$$\frac{dV}{dr} = 0 \quad \text{or} \quad V = \text{constant}$$

Hence electric potential is constant throughout the volume of a conductor and has the same value (as inside) on its surface. *Thus the surface of a conductor is an equipotential surface.*

If a conductor is charged, there exists an electric field normal to its surface. This indicates that the potential on the surface will be different from the potential at a point just outside the surface.

5. ***Electric field at the surface of a charged conductor is proportional to the surface charge density.*** Consider a charged conductor of irregular shape. Let σ be the surface charge density at any point of its surface. To

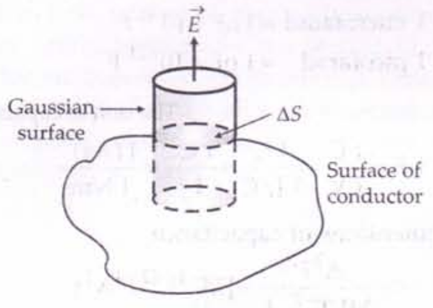


Fig. 2.42 A small pill box as a Gaussian surface of a charged conductor.

determine E at this point, we choose a short cylinder (pill box) as the Gaussian surface about this point. The pill box lies partly inside and partly outside the conductor. It has a cross-sectional area ΔS and negligible height.

Electric field is zero inside the conductor and just outside, it is normal to the surface. The contribution to the total flux through the pill box comes only from its outer cross-section.

$\therefore \quad \phi_E = E\,\Delta S$

Charge enclosed by pill box, $q = \sigma\,\Delta S$

By Gauss's theorem,

$$\phi_E = \frac{q}{\varepsilon_0}$$

$$\therefore \quad E\,\Delta S = \frac{\sigma\,\Delta S}{\varepsilon_0} \quad \text{or} \quad E = \frac{\sigma}{\varepsilon_0}$$

As $\vec{E}$ points normally outward, so we write

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

where $\hat{n}$ is a unit vector normal to the surface in the outer direction.

6. ***Electric field is zero in the cavity of a hollow charged conductor.*** As shown in Fig. 2.43, consider a charged conductor having a cavity, with no charges inside the cavity. Imagine a Gaussian surface inside the conductor quite close to the cavity. Everywhere inside the conductor, $E = 0$. By Gauss's theorem, charge enclosed by this Gaussian surface is zero ($E = 0 \Rightarrow q = 0$). Consequently, the electric field must be zero at every point inside the cavity ($q = 0 \Rightarrow E = 0$). The entire excess charge $+q$ lies on its surface.

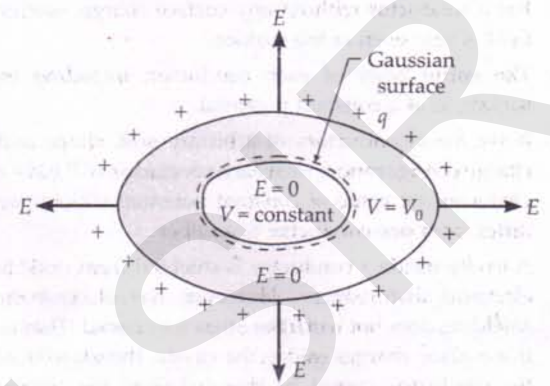


Fig. 2.43 Electric field vanishes in the cavity of a conductor.

2.16 ELECTROSTATIC SHIELDING

26. *What is electrostatic shielding ? Mention its few applications.*

Electrostatic shielding. Consider a conductor with a cavity, with no charges placed inside the cavity. Whatever be the size and shape of the cavity and whatever be the charge on the conductor and the external fields in which it might be placed, the electric field inside the cavity is zero, *i.e.*, the cavity inside the conductor remains shielded from outside electric influence. This is known as electrostatic shielding. Such a field free region is called a *Faraday cage*.

The phenomenon of making a region free from any electric field is called ***electrostatic shielding****. It is based on the fact that electric field vanishes inside the cavity of a hollow conductor.*

Applications of electrostatic shielding

1. In a thunderstorm accompanied by lightning, it is safest to sit inside a car, rather than near a tree or on the open ground. The metallic body of the car becomes an electrostatic shielding from lightning.
2. Sensitive components of electronic devices are protected or shielded from external electric disturbances by placing metal shields around them.
3. In a coaxial cable, the outer conductor connected to ground provides an electrical shield to the signals carried by the central conductor.

For Your Knowledge

- In the interior of a conductor, the electric field and the volume charge density both vanish. Therefore, charges in a conductor can only be at the surface.
- Electric field at the surface of a charged conductor must be normal to the surface at every point.
- For a conductor without any surface charge, electric field is zero even at the surface.
- The entire body of each conductor, including its surface, is at a constant potential.
- If we have conductors of arbitrary size, shape and charge configuration, then each conductor will have a characteristic value of constant potential which may differ from one conductor to another.
- A cavity inside a conductor is shielded from outside electrical disturbances. However, the electrostatic shielding does not work the other way round. That is, if we place charges inside the cavity, the exterior of the conductor cannot be shielded from the electric fields of the inside charges.

2.17 ELECTRICAL CAPACITANCE OF A CONDUCTOR

27. *Define electrical capacitance of a conductor. On which factors does it depend ?*

Electrical capacitance of a conductor. *The electrical capacitance of a conductor is the measure of its ability to hold electric charge.* When an insulated conductor is given some charge, it acquires a certain potential. If we increase the charge on a conductor, its potential also increases. If a charge Q put on an insulated conductor increases its potential by V, then

$$Q \propto V \quad \text{or} \quad Q = CV$$

The proportionality constant C is called the capacitance of the conductor. Thus

$$\text{Capacitance} = \frac{\text{Charge}}{\text{Potential}}$$

Hence the ***capacitance*** *of a conductor may be defined as the charge required to increase the potential of the conductor by unit amount.*

The capacitance of a conductor is the measure of its capacity to hold a large amount of charge without running a high potential. It depends upon the following factors :

1. Size and shape of the conductor.
2. Nature (permittivity) of the surrounding medium.
3. Presence of the other conductors in its neighbourhood.

It is worth-noting that the capacitance of a conductor does not depend on the nature of its material and the amount of charge existing on the conductor.

28. *Define the unit of capacitance for a conductor. Give its dimensions.*

Units of capacitance. The SI unit of capacitance is *farad* (F), named in the honour of *Michael Faraday.*

The capacitance of conductor is 1 **farad** *if the addition of a charge of 1 coulomb to it, increases its potential by 1 volt.*

$$\therefore\ 1\text{ farad} = \frac{1\text{ coulomb}}{1\text{ volt}} \quad \text{or} \quad 1\text{ F} = \frac{1\text{ C}}{1\text{ V}} = 1\text{ CV}^{-1}$$

One farad is a very large unit of capacitance. For practical purposes, we use its following submultiples :

$$1\text{ millifarad} = 1\text{ mF} = 10^{-3}\text{ F}$$
$$1\text{ microfarad} = 1\,\mu\text{F} = 10^{-6}\text{ F}$$
$$1\text{ picofarad} = 1\text{ pF} = 10^{-12}\text{ F}$$

Dimensions of capacitance. The unit of capacitance is

$$1\text{ F} = \frac{1\text{ C}}{1\text{ V}} = \frac{1\text{ C}}{1\text{ J/C}} = \frac{1\text{ C}^2}{1\text{ J}} = \frac{1\,(\text{As})^2}{1\text{ Nm}}$$

$\therefore$ Dimensions of capacitance

$$= \frac{\text{A}^2\text{T}^2}{\text{MLT}^{-2}.\text{L}} = [\text{M}^{-1}\text{L}^{-2}\text{T}^4\text{A}^2]$$

2.18 CAPACITANCE OF AN ISOLATED SPHERICAL CAPACITOR

29. *Obtain an expression for the capacitance of an isolated spherical conductor of radius R.*

Capacitance of an isolated spherical conductor. Consider an isolated spherical conductor of radius R. The charge Q is uniformly distributed over its entire surface. It can be assumed to be concentrated at the centre of the sphere. The potential at any point on the surface of the spherical conductor will be

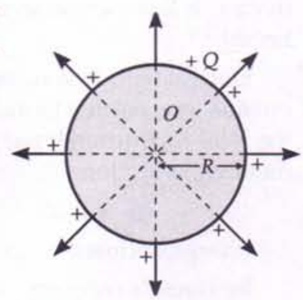


Fig. 2.44 Capacitance of a spherical conductor.

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R}$$

$\therefore$ Capacitance of the spherical conductor situated in vacuum is

$$C = \frac{Q}{V} = \frac{Q}{\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R}}$$

or $$C = 4\pi\varepsilon_0 R$$

Clearly, the *capacitance of a spherical conductor is proportional to its radius.*

Let us calculate the radius of the spherical conductor of capacitance 1 F.

$$R = \frac{1}{4\pi\varepsilon_0} \cdot C = 9 \times 10^9 \text{ mF}^{-1} \cdot 1 \text{ F}$$
$$= 9 \times 10^9 \text{m} = 9 \times 10^6 \text{km}$$

This radius is about 1500 times the radius of the earth ($\sim 6 \times 10^3$ km). So we conclude :

1. One farad is a very large unit of capacitance.
2. It is not possible to have a single isolated conductor of very large capacitance.

For Your Knowledge

- The formula : $C = 4\pi\varepsilon_0 R$ is valid for both hollow and solid spherical conductors.

 As $\varepsilon_0 = \frac{C}{4\pi R}$

 So the SI unit of ε_0 can be written as *farad per metre* (Fm^{-1}). From Coulomb's law, the SI unit of ε_0 comes out to be $\text{C}^2\text{N}^{-1}\text{m}^{-2}$. Both of these units are equivalent.
- The farad ($1\text{F} = 1 \text{ CV}^{-1}$) is an enormously large unit of capacitance because the coulomb is a very big unit of charge while the volt is the unit of potential having reasonable size.

Examples Based on Capacitance of Spherical Conductors

Formulae Used

1. Capacitance of a spherical conductor of radius R,
 $$C = 4\pi\varepsilon_0 R$$
2. Capacitance $= \frac{\text{Charge}}{\text{Potential}}$ or $C = \frac{q}{V}$.

Units Used

Charge is in coulomb, potential in volt and capacitance in farad (F).

Example 32. *An isolated sphere has a capacitance 50 pF. (i) Calculate its radius. (ii) How much charge should be placed on it to raise its potential to 10^4 V ?*

Solution. Here $C = 50 \text{ pF} = 50 \times 10^{-12}$ F, $V = 10^4$ V

(*i*) $R = \frac{1}{4\pi\varepsilon_0} \cdot C = 9 \times 10^9 \text{mF}^{-1} \times 50 \times 10^{-12} \text{ F}$

$= 45 \times 10^{-2} \text{m} = \mathbf{45 \text{ cm.}}$

(*ii*) $q = CV = 50 \times 10^{-12} \times 10^4 = 5 \times 10^{-7} \text{C} = \mathbf{0.5\ \mu C.}$

Example 33. *Twenty seven spherical drops of radius 3 mm and carrying 10^{-12} C of charge are combined to form a single drop. Find the capacitance and the potential of the bigger drop.* [Haryana 01]

Solution. Let r and R be the radii of the small and bigger drops, respectively.

Volume of the bigger drop

$= 27 \times$ Volume of a small drop

i.e., $\frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$

or $R = 3r = 3 \times 3 \text{ mm} = 9 \times 10^{-3}$ m

∴ Capacitance of the bigger drop is

$$C = 4\pi\varepsilon_0 R = \frac{1}{9 \times 10^9} \cdot 9 \times 10^{-3} \text{ F}$$
$$= 10^{-12} \text{ F} = \mathbf{1 \text{ pF}}$$

Charge on bigger drop

$$q = 27 \times \text{Charge on a small drop}$$
$$= 27 \times 10^{-12} \text{ C}$$

∴ Potential of bigger drop is

$$V = \frac{q}{C} = \frac{27 \times 10^{-12}}{10^{-12}} = \mathbf{27 \text{ V.}}$$

Example 34. *Eight identical spherical drops, each carrying a charge 1 nC are at a potential of 900 V each. All these drops combine together to form a single large drop. Calculate the potential of this large drop. (Assume no wastage of any kind and take the capacitance of a sphere of radius r as proportional to r).* [CBSE Sample Paper 15]

Solution.

Capacitance of each small drop, $C \propto r \Rightarrow C = kr$

Charge on each small drop, $q = CV = (kr \times 900)$C

Charge on large drop, $q' = 8q = 7200kr$ C

Volume of a large drop = Volume of 8 small drops

$$\frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3 \quad \Rightarrow \quad R = 8^{1/3} r = 2r$$

Capacitance of large drop, $C' = kR = 2kr$

Hence, the potential of the large drop is

$$V' = \frac{q'}{C'} = \frac{7200kr}{2kr} = \mathbf{3600 \text{ V.}}$$

Example 35. *A charged spherical conductor has a surface charge density of 0.07 C cm^{-2}. When the charge is increased by 4.4 C, the surface charge density changes by 0.084 C cm^{-2}. Find the initial charge and capacitance of the spherical conductor.*

Solution. Let q be the charge on the spherical conductor and r its radius. Its surface charge density is

$$\frac{q}{4\pi r^2} = 0.07 \text{ C cm}^{-2} \qquad ...(i)$$

When the charge is increased by 4.4 C, the surface charge density becomes

$$\frac{q + 4.4}{4\pi r^2} = 0.084 \text{ C cm}^{-2} \qquad ...(ii)$$

Dividing equation (*ii*) by (*i*), we get

$$\frac{q+4.4}{q}=\frac{0.084}{0.07} \quad \text{or} \quad q = \mathbf{22\ C}$$

From equation (*i*), we get

$$r=\sqrt{\frac{q}{4\pi\times 0.07}}=\sqrt{\frac{22\times 7}{4\times 22\times 0.07}}=5\text{ cm}=0.05\text{ m}$$

Capacitance,

$$C=4\pi\,\varepsilon_0\,r=\frac{1}{9\times 10^9}\times 0.05=\mathbf{5.56\times 10^{-12}\ F}.$$

Problems For Practice

1. Find the capacitance of a conducting sphere of radius 10 cm situated in air. How much charge is required to raise it to a potential of 1000 volt ?
(Ans. 11 pF, 1.1×10^{-8} C)

2. Assuming the earth to be a spherical conductor of radius 6400 km, calculate its capacitance.
[Himachal 98C ; Haryana 98C]
(Ans. 711 μF)

3. *N* drops of mercury of equal radii and possessing equal charges combine to form a big drop. Compare the charge, capacitance and potential of bigger drop with the corresponding quantities of individual drops.
[Punjab 01]
(Ans. N, $N^{1/3}$, $N^{2/3}$)

HINTS

1. $C=4\pi\,\varepsilon_0 R=\dfrac{1}{9\times 10^9}\times 0.10=11\times 10^{-12}\text{ F}=\mathbf{11\ pF}.$

   $q=CV=11\times 10^{-12}\text{ F}\times 1000\text{ V}=\mathbf{1.1\times 10^{-8}\ C}.$

2. $C=4\pi\,\varepsilon_0 R=\dfrac{1}{9\times 10^9}\times 6.4\times 10^6$

   $=0.711\times 10^{-3}\text{ F}=\mathbf{711\ \mu F}.$

3. Let q be the charge on each small drop and r its radius.

   Capacitance of each small drop, $C=4\pi\,\varepsilon_0\,r$

   Potential of each small drop, $V=\dfrac{1}{4\pi\,\varepsilon_0}\cdot\dfrac{q}{r}$

   If R is the radius of the big drop, then

   $$\frac{4}{3}\pi r^3\times N=\frac{4}{3}\pi R^3 \quad \text{or} \quad R=N^{1/3}\,r$$

   Charge on the big drop, $q'=Nq$ or $\dfrac{q'}{q}=\mathbf{N}.$

   Capacitance of the big drop,

   $$C'=4\pi\,\varepsilon_0\,R=4\pi\,\varepsilon_0\,N^{1/3}\,r=N^{1/3}\,C$$

   or $\dfrac{C'}{C}=\mathbf{N^{1/3}}.$

   Potential of the big drop,

   $$V'=\frac{1}{4\pi\,\varepsilon_0}\cdot\frac{q'}{R}=\frac{1}{4\pi\,\varepsilon_0}\cdot\frac{Nq}{N^{1/3}\,r}$$

   $$=N^{2/3}\cdot\frac{1}{4\pi\,\varepsilon_0}\cdot\frac{q}{r}=N^{2/3}\,V$$

   or $\dfrac{V'}{V}=N^{2/3}.$

2.19 CONCEPT OF A CAPACITOR AND ITS PRINCIPLE

30. *An isolated conductor cannot have a large capacitance, why ?*

The capacitance of an isolated conductor is small. When a conductor holds a large amount of charge, its potential is also high. If the associated electric field ($E=\sigma/\varepsilon_0$) becomes high enough, the atoms or molecules of the surrounding air get ionised. A *breakdown* occurs in the insulation of the surrounding medium and the charge put on the conductor gets neutralised or leaks away. For air, the breakdown point occurs at fields of the order of $3\times 10^6\text{ Vm}^{-1}$. This puts the limit on the capacitance of a conductor. Moreover, if we tend to have a single conductor of large capacitance, it will have practically inconvenient large size.

31. *Why does the capacitance of a conductor increase, when an earthed connected conductor is placed near it ? Briefly explain.*

Principle of a capacitor. Consider a positively charged metal plate *A* and place an uncharged plate *B* close to it, as shown in Fig. 2.45. Due to induction, the closer face of plate *B* acquires negative charge and its farther face acquires a positive charge. The negative charge on plate *B* tends to reduce the potential on plate *A*, while the positive charge on plate *B* tends to increase the potential on *A*. As the negative charge of plate *B* is closer to plate *A* than its positive charge, so the net effect is that the potential of *A* decreases by a small amount and hence its capacitance increases by a small amount.

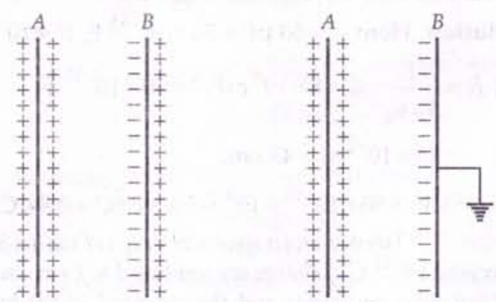


Fig. 2.45 Principle of a capacitor.

Now if the positive face of plate B is earthed, its positive charge gets neutralised due to the flow of electrons from the earth to the plate B. The negative charge on B is held in position due to the positive charge on A. The negative charge on B reduces the potential of A considerably and hence increases its capacitance by a large amount.

Hence we see that *the capacitance of an insulated conductor is considerably increased when we place an earthed connected conductor near it*. Such a system of two conductors is called a ***capacitor***.

32. *What is a capacitor ? Define capacitance of a capacitor. On what factors does it depend ?*

Capacitor. *A capacitor is an arrangement of two conductors separated by an insulating medium that is used to store electric charge and electric energy.*

A capacitor, in general, consists of two conductors of any size and shape carrying different potentials and charges, and placed closed together in some definite positions relative to one another.

Pictorial representation of a capacitor. The pictorial symbol for a capacitor with fixed capacitance is as shown in Fig. 2.46(*a*) and for that with a variable capacitance is as shown in Fig. 2.46(*b*).

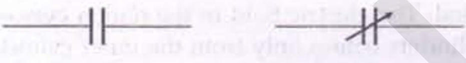


Fig. 2.46 Symbols for a capacitor with (*a*) fixed, (*b*) variable capacitance.

Capacitance of a capacitor. As shown in Fig. 2.47, usually a capacitor consists of two conductors having charges $+Q$ and $-Q$. The potential difference between them is $V = V_+ - V_-$. Here Q is called the charge on the capacitor. Note that the *charge on capacitor* does not mean the total charge given to the capacitor which is $+Q - Q = 0$.

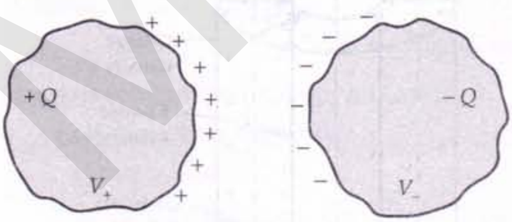


Fig. 2.47 Two conductors separated by an insulator form a capacitor.

For a given capacitor, the charge Q on the capacitor is proportional to the potential difference V between the two conductors. Thus,

$$Q \propto V \quad \text{or} \quad Q = CV$$

The proportionality constant C is called the *capacitance of the capacitor*. Clearly,

$$C = \frac{Q}{V}$$

$$\text{or} \quad \text{Capacitance} = \frac{\text{Charge on either conductor}}{\text{P.D. between the two conductors}}$$

The *capacitance of a capacitor may be defined as the charge required to be supplied to either of the conductors of the capacitor so as to increase the potential difference between them by unit amount.*

The capacitance of a given capacitor is a constant and depends on the *geometric factors*, such as the shapes, sizes and relative positions of the two conductors, and the nature of the medium between them.

SI unit of capacitance is farad (F). *A capacitor has a capacitance of 1 farad if 1 coulomb of charge is transferred from its one conductor to another on applying a potential difference of 1 volt across the two conductors.*

2.20 PARALLEL PLATE CAPACITOR

33. *What is a parallel plate capacitor ? Drive an expression for its capacitance. On what factors does the capacitance of a parallel plate capacitor depend ?*

Parallel plate capacitor. The simplest and the most widely used capacitor is the parallel plate capacitor. *It consists of two large plane parallel conducting plates, separated by a small distance.*

Let A = area of each plate,

d = distance between the two plates

$\pm\sigma$ = uniform surface charge densities on the two plates

$\pm Q = \pm\sigma A$ = total charge on each plate.

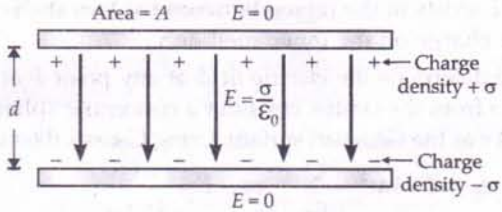


Fig. 2.48 Parallel plate capacitor.

In the outer regions above the upper plate and below the lower plate, the electric fields due to the two charged plates cancel out. The net field is zero.

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

In the inner region between the two capacitor plates, the electric fields due to the two charged plates add up. The net field is

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

The direction of the electric field is from the positive to the negative plate and the field is uniform throughout. For plates with finite area, the field lines bend at the edges. This effect is called ***fringing of the field***. But for large plates separated by small distance ($A >> d^2$), the field is almost uniform in the regions far from the edges. For a uniform electric field,

P.D. between the plates

= Electric field × distance between the plates

or $V = Ed = \frac{\sigma d}{\varepsilon_0}$

Capacitance of the parallel plate capacitor is

$$C = \frac{Q}{V} = \frac{\sigma A}{\sigma d / \varepsilon_0} \quad \text{or} \quad C = \frac{\varepsilon_0 A}{d}$$

Factors on which the capacitance of a parallel plate capacitor depends

1. Area of the plates ($C \propto A$).
2. Distance between the plates ($C \propto 1/d$).
3. Permittivity of the medium between the plates ($C \propto \varepsilon$).

2.21 SPHERICAL CAPACITOR*

34. *What is a spherical capacitor ? Derive an expression for its capacitance.*

Spherical capacitor. *A spherical capacitor consists of two concentric spherical shells of inner and outer radii a and b.* The two shells carry charges $-Q$ and $+Q$ respectively. Since the electric field inside a hollow conductor is zero, so $\vec{E} = 0$ for $r < a$. Also the field is zero outside the outer shell, *i.e.*, $\vec{E} = 0$ for $r > b$. A radial field $\vec{E}$ exists in the region between the two shells due to the charge on the inner shell only.

To determine the electric field at any point P at distance r from the centre, consider a concentric sphere of radius r as the Gaussian surface. Using Gauss's theorem,

$$\phi_E = E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \quad \text{or} \quad E = \frac{Q}{4\pi \varepsilon_0 r^2}$$

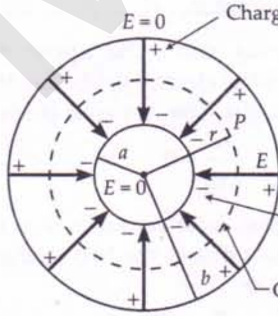


Fig. 2.49 Spherical capacitor.

The potential difference (caused by the inner sphere alone) between the two shells will be

$$V = -\int_a^b \vec{E} \cdot \vec{dr} = \int_a^b E dr = \int_a^b \frac{Q}{4\pi \varepsilon_0 r^2} dr$$

$$= \frac{Q}{4\pi \varepsilon_0} \int_a^b r^{-2} dr = \frac{Q}{4\pi \varepsilon_0} \left[-\frac{1}{r} \right]_a^b = \frac{Q}{4\pi \varepsilon_0} \left[\frac{1}{a} - \frac{1}{b} \right]$$

[∵ $\vec{E}$ points radially inward and $\vec{dr}$ points outward so $\vec{E} \cdot d\vec{r} = Edr\, 180° = -Edr$]

The capacitance of the spherical capacitor is

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi \varepsilon_0} \left[\frac{1}{a} - \frac{1}{b} \right]} \quad \text{or} \quad C = \frac{4\pi \varepsilon_0 ab}{b - a}.$$

2.22 CYLINDRICAL CAPACITOR*

35. *What is a cylindrical capacitor ? Derive an expression for its capacitance.*

Cylindrical capacitor. *A cylindrical capacitor consists of two coaxial conducting cylinders of inner and outer radii a and b.* Let the two cylinders have uniform linear charge densities of $\pm \lambda$ Cm^{-1}. The length L of the capacitor is so large ($L >>$ radii a or b) that the edge effect can be neglected. The electric field in the region between the two cylinders comes only from the inner cylinder, the outer cylinder does not contribute due to shielding. To calculate the electric field E at any point P in between the two cylinders at a distance r from the central axis, we consider a coaxial Gaussian cylinder of radius r. Using Gauss's theorem, the flux through Gaussian surface must be

$$\phi_E = \frac{q}{\varepsilon_0}$$

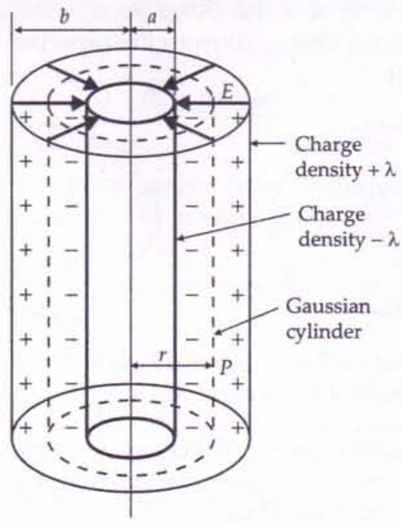


Fig. 2.50

or $\quad E \cdot 2\pi r\, L = \dfrac{\lambda L}{\varepsilon_0}$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$\therefore$ Potential difference between the two cylinders is

$$V = -\int_a^b \vec{E} \cdot \vec{dr} = \int_a^b E\,dr \qquad [\because \vec{E} \text{ and } d\vec{r} \text{ are in opposite directions}]$$

$$= \int_a^b \frac{\lambda}{2\pi\varepsilon_0 r}\,dr = \frac{\lambda}{2\pi\varepsilon_0}\int_a^b \frac{1}{r}\,dr$$

$$= \frac{\lambda}{2\pi\varepsilon_0}[\ln r]_a^b = \frac{\lambda}{2\pi\varepsilon_0}[\ln b - \ln a]$$

or $\quad V = \dfrac{\lambda}{2\pi\varepsilon_0}\ln\dfrac{b}{a}$

Total charge on each cylinder is $\quad Q = L\lambda$

$\therefore$ Capacitance of cylindrical capacitor is

$$C = \frac{Q}{V} = \frac{L\lambda}{\dfrac{\lambda}{2\pi\varepsilon_0}\ln\dfrac{b}{a}} \quad \text{or} \quad C = \frac{2\pi\varepsilon_0 L}{\ln\dfrac{b}{a}}$$

Examples based on
Capacitance of Air-Filled Capacitors

Formulae Used

1. Capacitance, $\quad C = \dfrac{q}{V}$
2. Capacitance of a parallel plate capacitor, $C = \dfrac{\varepsilon_0 A}{d}$
3. P.D. between the two plates of a capacitor having charges q_1 and q_2,
$$V = \frac{q_1 - q_2}{2C}$$
4. Capacitance of a spherical capacitor, $C = 4\pi\varepsilon_0 \dfrac{ab}{b-a}$

   Here a and b are the radii of inner and outer shells of the spherical capacitor.
5. Capacitance of a cylindrical capacitor,
$$C = 2\pi\varepsilon_0 \frac{L}{\log_e \frac{b}{a}} = 2\pi\varepsilon_0 \frac{L}{2.303\log_{10}\frac{b}{a}}$$
   Here a and b are the radii of inner and outer coaxial cylinders and L is the length of the capacitor.

Units Used

Capacitance C is in farad, charge q in coulomb, potential difference V in volt, thicknesses d and t in metre.

Constant Used

Permittivity constant, $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$

Example 36. *When 1.0×10^{12} electrons are transferred from one conductor to another of a capacitor, a potential difference of 10 V develops between the two conductors. Calculate the capacitance of the capacitor.*

Solution. Here $q = ne = 1.0 \times 10^{12} \times 1.6 \times 10^{-19}$

$$= 1.6 \times 10^{-7}\ \text{C}$$
$$V = 10\ \text{V}$$

$$\therefore \quad C = \frac{q}{V} = \frac{1.6 \times 10^{-7}}{10} = \mathbf{1.6 \times 10^{-8}\ F.}$$

Example 37. *A capacitor of unknown capacitance is connected across a battery of V volts. The charge stored in it is 360 μC. When potential across the capacitor is reduced by 120 V, the charge stored in it becomes 120 μC. Calculate :*

(i) The potential V and the unknown capacitance C.

(ii) What will be the charge stored in the capacitor, if the voltage applied had increased by 120 V ?

[CBSE D 13]

Solution. (*i*) Let C be the capacitance of the capacitor and V the potential drop across the plates. Then

$$q = CV = 360\ \mu\text{C}$$

When the potential difference is reduced by 120 V,

$$q' = C(V - 120) = 120\ \mu\text{C}$$

$$\therefore \quad \frac{V}{V-120} = \frac{360}{120} = 3 \quad \Rightarrow \quad V = \mathbf{180\ V}$$

$$C = \frac{q}{V} = \frac{360\ \mu\text{C}}{180\ \text{V}} = \mathbf{2\,\mu F.}$$

(*ii*) When the voltage is increased by 120 V,

$$q'' = C(V + 120) = 2\,\mu\text{F} \times (180 + 120) = \mathbf{600\,\mu C.}$$

Example 38. *A parallel plate capacitor has plate area of $25.0\ cm^2$ and a separation of 2.0 mm between its plates. The capacitor is connected to 12 V battery. (i) Find the charge on the capacitor. (ii) If the plate separation is decreased by 1.0 mm, what extra charge is given by the battery to the positive plate ?*

Solution. $A = 25.0\ \text{cm}^2 = 25 \times 10^{-4}\ \text{m}^2$,

$$d = 2.0\ \text{mm} = 2 \times 10^{-3}\ \text{m},\ V = 12\ \text{V}$$

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 25 \times 10^{-4}}{2 \times 10^{-3}} = 1.1 \times 10^{-11}\ \text{F}$$

(*i*) $q = CV = 1.1 \times 10^{-11} \times 12 = \mathbf{1.32 \times 10^{-10}\ C}$

(*ii*) Here $d' = 2.0 - 1.0 = 1.0\ \text{mm} = 1 \times 10^{-3}\ \text{m}$

$$\therefore \quad C' = \frac{8.85 \times 10^{-12} \times 25 \times 10^{-4}}{1 \times 10^{-3}} = 2.2 \times 10^{-11}\ \text{F}$$

$$q' = C'V = 2.2 \times 10^{-11} \times 12 = 2.64 \times 10^{-10}\ \text{C}$$

Extra charge given by the battery to the positive plate is

$$q' - q = (2.64 - 1.32) \times 10^{-10} = \mathbf{1.32 \times 10^{-10}\ C.}$$

Example 39. *Two parallel plate air capacitors have their plate areas 100 and 500 cm^2 respectively. If they have the same charge and potential and the distance between the plates of the first capacitor is 0.5 mm, what is the distance between the plates of the second capacitor ?* [Punjab 97C]

Solution. As capacitance, $C = q/V$ and the two capacitors have the same charge q and potential V, so they have the equal capacitances, *i.e.*,

$$C_1 = C_2$$

or $$\frac{\varepsilon_0 A_1}{d_1} = \frac{\varepsilon_0 A_2}{d_2}$$

or $$d_2 = \frac{A_2}{A_1} d_1$$

But $A_1 = 100 \text{ cm}^2$, $A_2 = 500 \text{ cm}^2$,

$d_1 = 0.5 \text{ mm} = 0.05 \text{ cm}$

$$\therefore \quad d_2 = \frac{500 \times 0.05}{100} = 0.25 \text{ cm} = \mathbf{2.5\ mm}.$$

Example 40. *A sphere of radius 0.03 m is suspended within a hollow sphere of radius 0.05 m. If the inner sphere is charged to a potential of 1500 volt and outer sphere is earthed, find the capacitance and the charge on the inner sphere.*

Solution. Here $a = 0.03$ m, $b = 0.05$ m, $V = 1500$ V

The capacitance of the air-filled spherical capacitor is

$$C = \frac{4\pi\varepsilon_0\, ab}{b-a} = \frac{0.03 \times 0.05}{9 \times 10^9 \times (0.05 - 0.03)}$$

$$= 8.33 \times 10^{-12} \text{ F} = \mathbf{8.33\ pF.}$$

Charge, $q = CV = 8.33 \times 10^{-12} \times 1500$

$= \mathbf{1.25 \times 10^{-8}\ C.}$

Example 41. *The thickness of air layer between the two coatings of a spherical capacitor is 2 cm. The capacitor has the same capacitance as the sphere of 1.2 m diameter. Find the radii of its surfaces.*

Solution. Here $\dfrac{4\pi\varepsilon_0\, ab}{b-a} = 4\pi\varepsilon_0 R$

or $$\frac{ab}{b-a} = R$$

Now $b - a = 2$ cm and $R = \dfrac{1.2}{2}$ m $= 60$ cm

$$\therefore \quad \frac{ab}{2} = 60$$

or $$ab = 120$$

$$(b+a)^2 = (b-a)^2 + 4ab$$

$$= 2^2 + 4 \times 120 = 484$$

or $$b + a = 22$$

or $$2 + a + a = 22 \qquad [\because b - a = 2 \text{ cm}]$$

$\therefore$ $a = \mathbf{10\ cm}$ and $b = \mathbf{12\ cm.}$

Example 42. *The negative plate of a parallel plate capacitor is given a charge of -20×10^{-8} C. Find the charges appearing on the four surfaces of the capacitor plates.*

Solution. As shown in Fig. 2.51, let the charge appearing on the inner surface of the negative plate be $-Q$. Then the charge on its outer surface will be $Q - 20 \times 10^{-8}$ C.

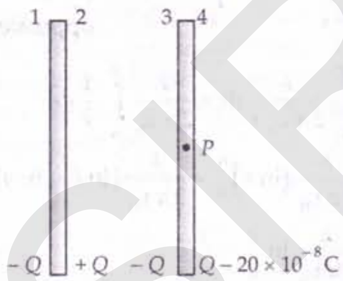


Fig. 2.51

The induced charge on the inner surface of the positive plate will be $+Q$ and that on the outer surface will be $-Q$, as the positive plate is electrically neutral. To find Q, we consider the electric field at a point P inside the negative plate.

Field due to surface 1 $= \dfrac{Q}{2\varepsilon_0 A}$, towards left

Field due to surface 2 $= \dfrac{Q}{2\varepsilon_0 A}$, towards right

Field due to surface 3 $= \dfrac{Q}{2\varepsilon_0 A}$, towards left

Field due to surface 4 $= \dfrac{20 \times 10^{-8}\text{ C}}{2\varepsilon_0 A}$, towards left

As the point P lies inside the conductor, the field here must be zero.

$$\frac{Q}{2\varepsilon_0 A} - \frac{Q}{2\varepsilon_0 A} + \frac{Q}{2\varepsilon_0 A} + \frac{Q - 20 \times 10^{-8}}{2\varepsilon_0 A} = 0$$

or $$2Q - 20 \times 10^{-8} = 0$$

$$Q = +10 \times 10^{-8} \text{ C}$$

$\therefore$ Charge on surface 1 = $\mathbf{-10 \times 10^{-8}}$ **C**

Charge on surface 2 = $\mathbf{+10 \times 10^{-8}}$ **C**

Charge on surface 3 = $\mathbf{-10 \times 10^{-8}}$ **C**

Charge on surface 4 = $\mathbf{-10 \times 10^{-8}}$ **C.**

Problems For Practice

1. A capacitor of 20 μF is charged to a potential of 10 kV. Find the charge accumulated on each plate of the capacitor. (Ans. 0.2 C)

2. Calculate the capacitance of a parallel plate capacitor having circular discs of radii 5.0 cm each. The separation between the discs is 1.0 mm. (Ans. 0.69×10^{-10} F)
3. A parallel plate air capacitor consists of two circular plates of diameter 8 cm. At what distance should the plates be held so as to have the same capacitance as that of a sphere of diameter 20 cm ? (Ans. 4 mm)
4. A parallel-plate capacitor has plates of area 200 cm^2 and separation between the plates 1.0 mm. (*i*) What potential difference will be developed if a charge of 1.0 nC is given to the capacitor ? (*ii*) If the plate separation is now increased to 2.0 mm, what will be the new potential difference ? (Ans. 5.65 V, 11.3 V)
5. Two metallic conductors have net charges of + 70 pC and − 70 pC, which result in a potential difference of 20 V between them. What is the capacitance of the system ? (Ans. 3.5 pF)
6. A spherical capacitor has an inner sphere of radius 9 cm and an outer sphere of radius 10 cm. The outer sphere is earthed and the inner sphere is charged. What is the capacitance of the capacitor ? (Ans. 0.1 nF)
7. The stratosphere acts as a conducting layer for the earth. If the stratosphere extends beyond 50 km from the surface of the earth, then calculate the capacitance of the spherical capacitor formed between stratosphere and earth's surface. Take radius of the earth as 6400 km. (Ans. 0.092 F)
8. A charge of $+2.0 \times 10^{-8}$ C is placed on the positive plate and a charge of -1.0×10^{-8} C on the negative plate of a parallel plate capacitor of capacitance 1.2×10^{-3} μF. Calculate the potential difference developed between the plates. (Ans. 12.5 V)

HINTS

1. $C = 20\,\mu F = 20 \times 10^{-6} F,\ V = 10\,kV = 10^4\,V$

   Charge, $q = CV = 20 \times 10^{-6} \times 10^4 C = \mathbf{0.2\ C}$.

2. Here $r = 5.0\,cm = 0.05\,m,\ d = 1.0\,mm = 10^{-3}\,m$

   Capacitance,
   $$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \pi r^2}{d}$$
   $$= \frac{\pi \times (0.05)^2}{4\pi \times 9 \times 10^9 \times 10^{-3}} = \mathbf{0.69 \times 10^{-10}\ F}.$$

3. $$\frac{\varepsilon_0 A}{d} = 4\pi \varepsilon_0 R \quad \text{or} \quad \frac{\varepsilon_0 \pi D^2}{4d} = 4\pi\varepsilon_0 R$$
   or $$d = \frac{D^2}{16R} = \frac{(0.08)^2}{16 \times 0.10} = 4 \times 10^{-3}\ m = \mathbf{4\ mm}.$$

4. $$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 200 \times 10^{-4}}{1 \times 10^{-3}}$$
   $$= 0.177 \times 10^{-9}\ F = 0.177\ nF$$
   (*i*) $$V = \frac{q}{C} = \frac{1\ nC}{0.177\ nF} = \mathbf{5.65\ V}.$$
   (*ii*) When the plate separation increases from 1.0 mm to 2.0 mm, the capacitance decreases by a factor of 2. For the same charge, the potential difference will increase by a factor of 2.

   $\therefore\ V' = 2\,V = 2 \times 5.65 = \mathbf{11.3\ V}$.

5. Charge on the capacitor,
   $$q = 70\ pC = 70 \times 10^{-12}\ C$$
   $$C = \frac{q}{V} = \frac{70 \times 10^{-12} C}{20\ V} = \mathbf{3.5\ pF}.$$

6. Here $a = 9\,cm = 0.09\,m,\ b = 10\,cm = 0.10\,m$
   $$C = \frac{4\pi\varepsilon_0 ab}{b-a} = \frac{1}{9 \times 10^9} \cdot \frac{0.09 \times 0.10}{(0.10 - 0.09)}\ F$$
   $$= \frac{0.01 \times 0.10}{0.01} \times 10^{-9} F = 0.1 \times 10^{-9} F = \mathbf{0.1\ nF}.$$

7. Here

   a = radius of the earth = 6.4×10^6 m

   b = distance of the stratosphere layer from the centre of the earth

   $= 6400 + 50 = 6450\ km = 6.45 \times 10^6$ m
   $$C = 4\pi\,\varepsilon_0 \frac{ab}{a-b} = \frac{1}{9 \times 10^9} \times \frac{6.4 \times 10^6 \times 6.45 \times 10^6}{(6.45 - 6.4) \times 10^6}$$
   $= \mathbf{0.092\ F}$.

8. $$V = \frac{q_1 - q_2}{2C} = \frac{2.0 \times 10^{-8} + 1.0 \times 10^{-8}}{2 \times 1.2 \times 10^{-9}} = \mathbf{12.5\ V}.$$

2.23 COMBINATION OF CAPACITORS IN SERIES AND IN PARALLEL

36. *A number of capacitors are connected in series. Derive an expression for the equivalent capacitance of the series combination.*

Capacitors in series. *When the negative plate of one capacitor is connected to the positive plate of the second, and the negative of the second to the positive of third and so on, the capacitors are said to be connected in series.*

Figure 2.52 shows three capacitors of capacitances C_1, C_2 and C_3 connected in series. A potential difference V is applied across the combination. This sets up charges $\pm Q$ on the two plates of each capacitor. What actually happens is, a charge $+Q$ is given to the left plate of capacitor C_1 during the charging process. The charge $+Q$ induces a charge $-Q$ on the right plate of C_1 and a charge $-Q$ on the left plate of C_2, etc.

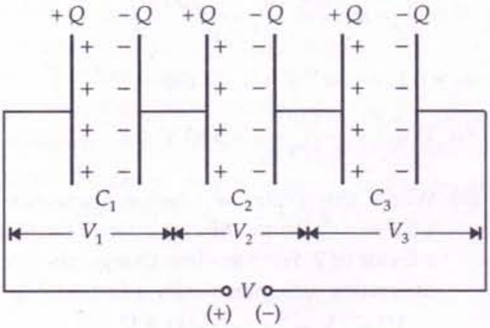


Fig. 2.52 Capacitors in series.

The potential differences across the various capacitors are

$$V_1 = \frac{Q}{C_1},\ V_2 = \frac{Q}{C_2},\ V_3 = \frac{Q}{C_3}$$

For the series circuit, the sum of these potential differences must be equal to the applied potential difference.

$$\therefore\quad V = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

or $$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \qquad ...(1)$$

Clearly, the combination can be regarded as an effective capacitor with charge Q and potential difference V. If C_s is the equivalent capacitance of the series combination, then

$$C_s = \frac{Q}{V}$$

or $$\frac{1}{C_s} = \frac{V}{Q} \qquad ...(2)$$

From equations (1) and (2), we get

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

For a series combination of n capacitors, we can write

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}$$

For series combination of capacitors

1. The reciprocal of equivalent capacitance is equal to the sum of the reciprocals of the individual capacitances.
2. The equivalent capacitance is smaller than the smallest individual capacitance.
3. The charge on each capacitors is same.
4. The potential difference across any capacitor is inversely proportional to its capacitance.

37. *A number of capacitors are connected in parallel. Derive an expression for the equivalent capacitance of the parallel combination.*

Capacitors in parallel. *When the positive plates of all capacitors are connected to one common point and the negative plates to another common point, the capacitors are said to be connected in parallel.*

Figure 2.53 shows three capacitors of capacitances C_1, C_2 and C_3 connected in parallel. A potential difference V is applied across the combination. All the capacitors have a common potential difference V but different charges given by

$$Q_1 = C_1V,\quad Q_2 = C_2V,\quad Q_3 = C_3V$$

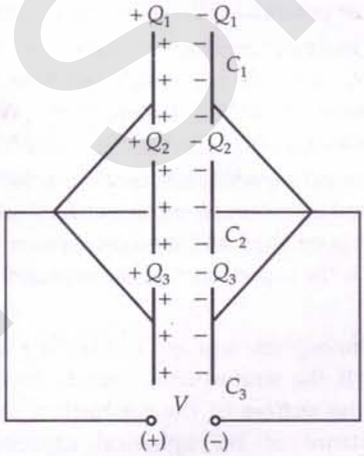


Fig. 2.53 Capacitors in parallel.

Total charge stored in the combination is

$$Q = Q_1 + Q_2 + Q_3 = (C_1 + C_2 + C_3)\,V \qquad ...(1)$$

If C_p is the equivalent capacitance of the parallel combination, then

$$Q = C_p\,V \qquad ...(2)$$

From equations (1) and (2), we get

$$C_pV = (C_1 + C_2 + C_3)\,V$$

or $$C_p = C_1 + C_2 + C_3$$

For a parallel combination of n capacitors, we can write

$$C_p = C_1 + C_2 + ... + C_n$$

For parallel combination of capacitors

1. The equivalent capacitance is equal to the sum of the individual capacitances.
2. The equivalent capacitance is larger than the largest individual capacitance.
3. The potential difference across each capacitor is same.
4. The charge on each capacitor is proportional to its capacitance.

Examples based on Grouping of Capacitors

Formulae Used

1. In series combination, $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$
2. In parallel combination, $C_p = C_1 + C_2 + C_3 + ...$
3. In series combination, charge on each capacitor is same (equal to the charge supplied by battery) but potential differences across the capacitors may be different.
4. In parallel combination, potential difference on each capacitor is same but the charges on the capacitors may be different.

Units Used

Capacitances are in farad, potential differences in volt and charges in coulomb.

Example 43. *Two capacitors of capacitance of 6 μF and 12 μF are connected in series with a battery. The voltage across the 6 μF capacitor is 2 V. Compute the total battery voltage.* [CBSE OD 06]

Solution. As the two capacitors are connected in series, the charge on each capacitor must be same.

∴ Charge on 6 μF capacitor = Charge on 12 μF capacitor

or $6\ \mu F \times 2$ volt $= 12\ \mu F \times V$ volt

∴ P.D. across 12 μF capacitor $= \frac{6 \times 2}{12} = 1$ volt

Battery voltage $= V_1 + V_2 = 2\text{ V} + 1\text{ V} = \mathbf{3\ V}$.

Example 44. *Two capacitors of capacitances 3 μF and 6 μF, are charged to potentials of 2 V and 5 V respectively. These two charged capacitors are connected in series. Find the potential across each of the two capacitors now.* **[CBSE Sample Paper 04]**

Solution. Total charge on the two capacitors

$$= C_1V_1 + C_2V_2 = (3 \times 2 + 6 \times 5)\ \mu C = 36\ \mu C$$

In series combination, charge is conserved.

∴ Charge on either capacitor,

$$q = 36\ \mu C$$

Potential on 3 μF capacitor $= \frac{q}{C_1} = \frac{36\ \mu C}{3\ \mu F} = \mathbf{12\ V}$

Potential on 6 μF capacitor $= \frac{q}{C_2} = \frac{36\ \mu C}{6\ \mu F} = \mathbf{6\ V}$.

Example 45. *Two capacitors have a capacitance of 5 μF when connected in parallel and 1.2 μF when connected in series. Calculate their capacitances.*

Solution. Let the two capacitances be C_1 μF and C_2 μF.

In parallel, $C_p = C_1 + C_2 = 5\ \mu F$

In series, $C_s = \frac{C_1C_2}{C_1 + C_2} = 1.2\ \mu F$

or $\frac{C_1(5 - C_1)}{5} = 1.2$

or $C_1^2 - 5C_1 + 6 = 0$

Hence, $C_1 = 2$ or $3\ \mu F$

∴ The capacitances are of **2 μF and 3 μF**.

Example 46. *Three capacitors of equal capacitance, when connected in series have net capacitance C_1, and when connected in parallel have net capacitance C_2. What is the value of C_1 / C_2?*

Solution. Let C = capacitance of each capacitor.

For series combination,

$$\frac{1}{C_1} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = \frac{3}{C} \quad \text{or} \quad C_1 = \frac{C}{3}$$

For parallel combination,

$$C_2 = C + C + C = 3C \quad \therefore \quad \frac{C_1}{C_2} = \frac{C}{3} \cdot \frac{1}{3C} = \mathbf{\frac{1}{9}}.$$

Example 47. *In Fig. 2.54, each of the uncharged capacitors has a capacitance of 25 μF. What charge will flow through the meter M when the switch S is closed?*

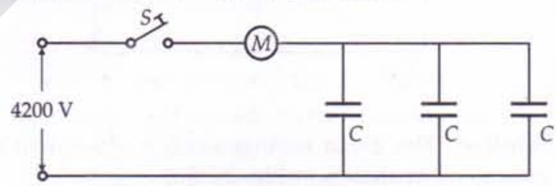


Fig. 2.54

Solution. As the three capacitors are connected in parallel, their equivalent capacitance is

$$C_p = C + C + C = 3C = 3 \times 25\ \mu F = 75\ \mu F$$
$$V = 4200\text{ V}$$

∴ Charge, $q = C_pV = 75 \times 10^{-6} \times 4200$

$= 315 \times 10^{-3}\text{ C} = \mathbf{315\ mC}$

Example 48. *Calculate the charge supplied by the battery in the arrangement shown in Fig. 2.55.*

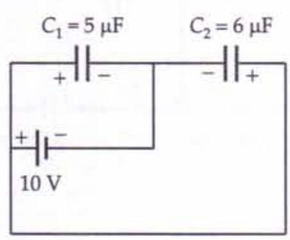


Fig. 2.55

Solution. The given arrangement is equivalent to the arrangement shown in Fig. 2.56.

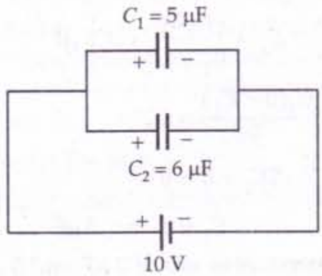


Fig. 2.56

Clearly, the two capacitors are connected in parallel. Their equivalent capacitance is

$$C = C_1 + C_2 = 5 + 6 = 11\ \mu F$$

Charge supplied by the battery is

$$q = CV = 11\ \mu F \times 10\ V = \mathbf{110\ \mu C}.$$

Example 49. *Three capacitors C_1, C_2 and C_3 are connected to a 6 V battery, as shown in Fig. 2.57. Find the charges on the three capacitors.*

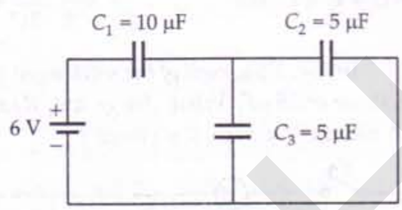


Fig. 2.57

Solution. The given arrangement is equivalent to the arrangement shown in Fig. 2.58(*a*).

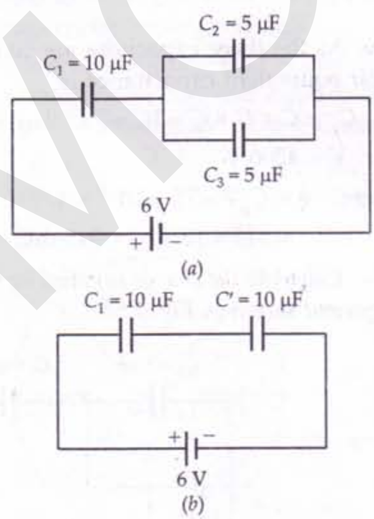


Fig. 2.58

Clearly, C_2 and C_3 are in parallel. Their equivalent capacitance is

$$C' = C_2 + C_3 = 5 + 5 = 10\ \mu F$$

Now C_1 and C' form a series combination, as shown in Fig. 2.58(*b*). Their equivalent capacitance is

$$C = \frac{C_1 C'}{C_1 + C'} = \frac{10 \times 10}{10 + 10} = 5\ \mu F$$

Charge drawn from the battery,

$$q = CV = 5\ \mu F \times 6\ V = 30\ \mu C$$

Charge on the capacitor $C_1 = q = \mathbf{30\ \mu C}$.

Charge on the parallel combination of C_2 and $C_3 = q = \mathbf{30\ \mu C}$

As C_2 and C_3 are equal, so the charge is shared equally by the two capacitors.

$$\text{Charge on } C_2 = \text{charge on } C_3 = \frac{30}{2} = \mathbf{15\ \mu C}.$$

Example 50. *Find the equivalent capacitance of the combination of capacitors between the points A and B as shown in Fig. 2.59. Also calculate the total charge that flows in the circuit when a 100 V battery is connected between the points A and B.* **[CBSE D 02]**

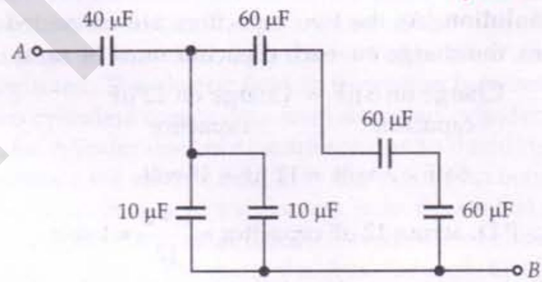


Fig. 2.59

Solution. Here three capacitors of 60 μF each are connected in series. Their equivalent capacitance C_1 is given by

$$\frac{1}{C_1} = \frac{1}{60} + \frac{1}{60} + \frac{1}{60} = \frac{3}{60} = \frac{1}{20}$$

or $\quad C = 20\ \mu F$

The given arrangement now reduces to the equivalent circuit shown in Fig. 2.60(*a*)

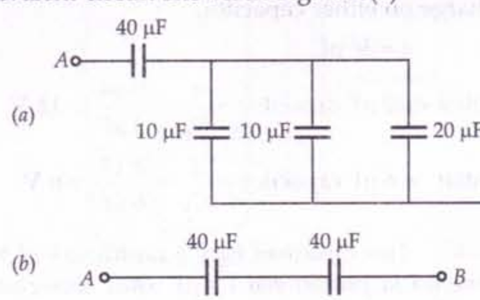


Fig. 2.60

Clearly, the three capacitors of 10 μF, 10 μF and 20 μF are in parallel. Their equivalent capacitance is

$$C_2 = 10 + 10 + 20 = 40\ \mu F$$

Now the circuit reduces to the equivalent circuit shown in Fig. 2.60(*b*). We have two capacitors of 40 μF each connected in series. The equivalent capacitance between *A* and *B* is

$$C = \frac{40 \times 40}{40 + 40} = \mathbf{20\ \mu F.}$$

Given $V = 100$ V

$\therefore$ Charge, $q = CV = 20\ \mu F \times 100\ V$

$$= 2000\ \mu C = \mathbf{2\ mC.}$$

Example 51. *If $C_1 = 3$ pF and $C_2 = 2$ pF, calculate the equivalent capacitance of the given network between points A and B.*

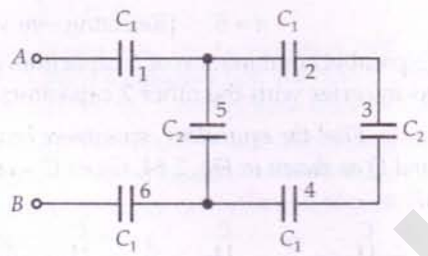


Fig. 2.61

Solution. Clearly, capacitors 2, 3 and 4 form a series combination. Their total capacitance C' is given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_1} = \frac{1}{3} + \frac{1}{2} + \frac{1}{3} = \frac{7}{6}$$

$$\therefore \quad C' = \frac{6}{7}\ pF$$

The capacitance C' forms a parallel combination with capacitor 5, so their equivalent capacitance is

$$C'' = C' + C_2 = \frac{6}{7} + 2 = \frac{20}{7}\ pF$$

The capacitance C'' forms a series combination with capacitors 1 and 6. The equivalent capacitance C of the entire network is given by

$$\frac{1}{C} = \frac{1}{C''} + \frac{1}{C_1} + \frac{1}{C_1} = \frac{7}{20} + \frac{1}{3} + \frac{1}{3} = \frac{61}{60}$$

$$\therefore \quad C = \mathbf{\frac{60}{61}\ pF.}$$

Example 52. *From the network shown in Fig. 2.62, find the value of the capacitance C if the equivalent capacitance between points A and B is to be 1 μF. All the capacitances are in μF.*

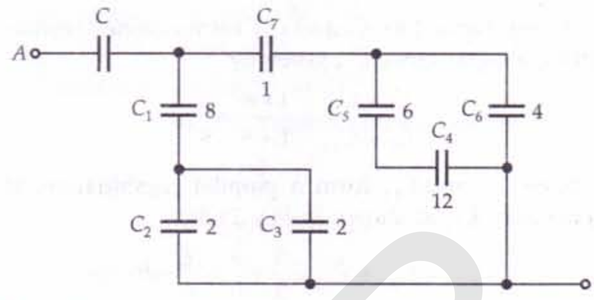


Fig. 2.62

Solution. Capacitors C_2 and C_3 form a parallel combination of equivalent capacitance,

$$C_8 = C_2 + C_3 = 2 + 2 = 4\ \mu F$$

Capacitors C_4 and C_5 form a series combination of capacitance C_9 given by

$$\frac{1}{C_9} = \frac{1}{C_4} + \frac{1}{C_5} = \frac{1}{12} + \frac{1}{6} = \frac{3}{12} = \frac{1}{4}$$

$$\therefore \quad C_9 = 4\ \mu F$$

The equivalent circuit can be shown as in Fig. 2.63(*a*)

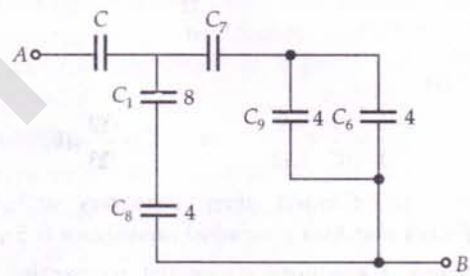


Fig. 2.63 (*a*)

Capacitors C_1 and C_8 form a series combination of capacitance C_{10} given by

$$C_{10} = \frac{C_1 C_8}{C_1 + C_8} = \frac{8 \times 4}{8 + 4} = \frac{32}{12} = \frac{8}{3}\ \mu F$$

Capacitors C_6 and C_9 form a parallel combination of capacitance.

$$C_{11} = C_6 + C_9 = 4 + 4 = 8\ \mu F$$

The given network reduces to the equivalent circuit Fig. 2.63(*b*).

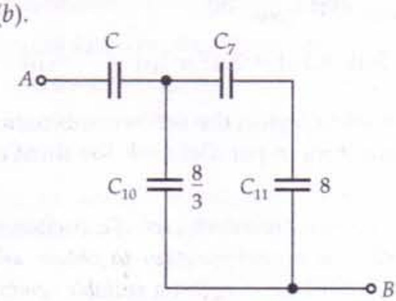


Fig. 2.63 (*b*)

Again, capacitors C_7 and C_{11} form a series combination of capacitance C_{12} given by

$$C_{12} = \frac{C_7 \times C_{11}}{C_7 + C_{11}} = \frac{1 \times 8}{1+8} = \frac{8}{9}\ \mu F$$

Now C_{10} and C_{12} form a parallel combination of capacitance C_{13} as shown in Fig. 2.63(c).

$$C_{13} = C_{10} + C_{12} = \frac{8}{3} + \frac{8}{9} = \frac{32}{9}\ \mu F$$

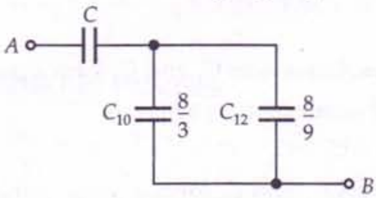


Fig. 2.63 (c)

Finally, the capacitors C and C_{13} form a series combination of capacitance 1 μF as shown in Fig. 2.63(d).

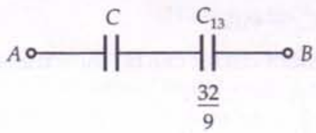


Fig. 2.63 (d)

$$\therefore \quad \frac{1}{1} = \frac{1}{C} + \frac{9}{32} \quad \text{or} \quad C = \frac{32}{23}\ \mu F.$$

Example 53. *Connect three capacitors of* 3 μF, 3 μF *and* 6 μF *such that their equivalent capacitance is* 5 μF.

Solution. Capacitors connected in parallel have maximum equivalent capacitance.

$$C_{max} = 3 + 3 + 6 = 12\ \mu F$$

Capacitors connected in series have minimum equivalent capacitance.

$$\frac{1}{C_{min}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{6} = \frac{5}{6}$$

or $$C_{min} = \frac{6}{5} = 1.2\ \mu F$$

The required equivalent capacitance of 5 μF lies between C_{max} and C_{min}. So

$$5\ \mu F = 3\ \mu F + 2\ \mu F = 3\ \mu F + \frac{3 \times 6}{3+6}\ \mu F$$

So we should connect the series combination of 3 μF and 6 μF capacitors in parallel with the third capacitor of 3 μF.

Example 54. *Seven capacitors, each of capacitance* 2 μF *are to be connected in a configuration to obtain an effective capacitance of* 10 / 11 μF. *Suggest a suitable combination to achieve the desired result.* [IIT 90]

Solution. Suppose a parallel combination of n capacitors is connected in series with a series combination of $(7 - n)$ capacitors.

Capacitance of parallel combination, $C_1 = 2n\ \mu F$

Capacitance of series combination, $C_2 = \frac{2}{7-n}\ \mu F$

As these two combinations are in series, so

$$C_s = \frac{10}{11}\ \mu F$$

But $$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} \quad \therefore \quad \frac{11}{10} = \frac{1}{2n} + \frac{7-n}{2}$$

Multiplying both sides by 10 n, we get

$$11n = 5 + 35n - 5n^2$$

or $$5n^2 - 24n - 5 = 0$$

$$(n-5)(5n+1) = 0$$

or $$n = 5 \quad \text{[Rejecting –ve value]}$$

Hence parallel combination of 5 capacitors must be connected in series with the other 2 capacitors.

Example 55. *Find the equivalent capacitance between the points P and Q as shown in Fig. 2.64. Given* $C = 18\ \mu F$ *and* $C_1 = 12\ \mu F$. [REC 97]

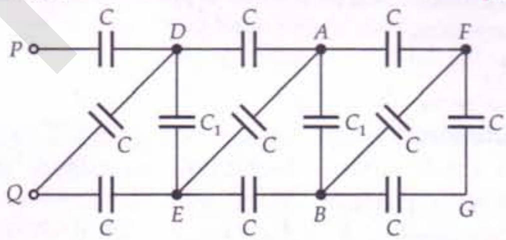


Fig. 2.64

Equivalent capacitance between points F and B is

$$\frac{18 \times 18}{18 + 18} + 18 = 27\ \mu F$$

Equivalent capacitance between points A and B is

$$12 + \frac{18 \times 27}{18 + 27} = 12 + 10.8 = 22.8 \simeq 23\ \mu F$$

Equivalent capacitance between points A and E is

$$\frac{23 \times 18}{23 + 18} + 18 = 28\ \mu F$$

Equivalent capacitance between points D and E is

$$\frac{28 \times 18}{28 + 18} + 12 = 23\ \mu F$$

Equivalent capacitance between points D and Q is

$$\frac{23 \times 18}{23 + 18} + 18 = 28\ \mu F$$

Equivalent capacitance between points P and Q is

$$\frac{28 \times 18}{28 + 18} = \mathbf{11\ \mu F.}$$

Example 56. *Four capacitors are connected as shown in the Fig. 2.65. Calculate the equivalent capacitance between the points X and Y.* [CBSE D 2000]

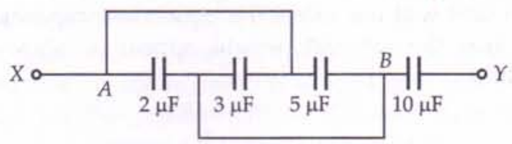


Fig. 2.65

Solution. Clearly, the first plate of 2 μF capacitor, the second plate of 3 μF capacitor and the first plate of 5 μF capacitor are connected to the point *A*. On the other hand, the second plate of 2 μF capacitor, the first plate of 3 μF capacitor and the second plate of 5 μF capacitor are connected to the point *B*. Thus the capacitors of 2 μF, 3 μF and 5 μF are connected in parallel between points *A* and *B*, as shown in the equivalent circuit diagram of Fig. 2.66.

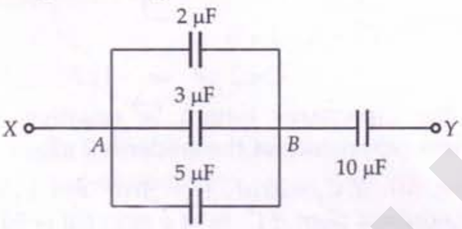


Fig. 2.66

Total capacitance of the parallel combination of capacitances 2 μF, 3 μF and 5 μF is

$$C' = 2 + 3 + 5 = 10\ \mu F$$

As shown in Fig. 2.67, this parallel combination is in series with capacitance of 10 μF.

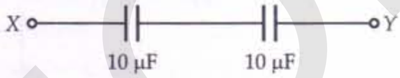


Fig. 2.67

Equivalent capacitance between X and Y

$$= \frac{10 \times 10}{10 + 10} = \mathbf{5\ \mu F.}$$

Example 57. *Five capacitors of capacitance 10 μF each are connected with each other, as shown in Fig. 2.68. Calculate the total capacitance between the points A and C.*

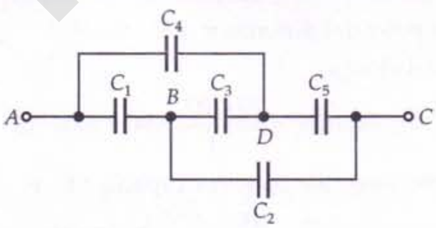


Fig. 2.68

Solution. The given circuit can be redrawn in the form of a wheatstone bridge as shown in Fig. 2.69.

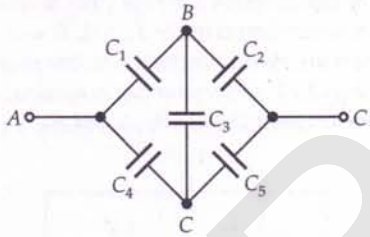


Fig. 2.69

As $C_1 = C_2 = C_4 = C_5$,

Therefore, $\frac{C_1}{C_2} = \frac{C_4}{C_5}$.

Thus the given circuit is a balanced wheatstone bridge. So the potential difference across the ends of capacitor C_3 is zero. Capacitance C_3 is ineffective. The given circuit reduces to the equivalent circuit shown in Fig. 2.70(*a*).

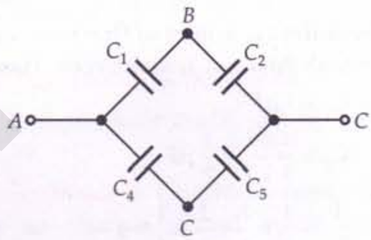


Fig. 2.70 (*a*)

Capacitors C_1 and C_2 form a series combination of equivalent capacitance C_6 given by

$$C_6 = \frac{C_1 \times C_2}{C_1 + C_2} = \frac{10 \times 10}{10 + 10} = 5\ \mu F$$

Similarly, C_4 and C_5 form a series combination of equivalent capacitance C_7 given by

$$C_7 = \frac{C_4 \times C_5}{C_4 + C_5} = \frac{10 \times 10}{10 + 10} = 5\ \mu F$$

As shown in Fig. 2.70(*b*), C_6 and C_7 form a parallel combination. Hence the equivalent capacitance of the network is given by

$$C = C_6 + C_7 = 5 + 5 = \mathbf{10\ \mu F.}$$

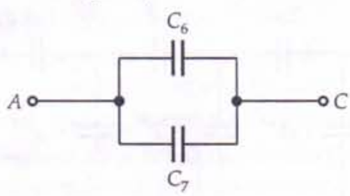


Fig. 2.70 (*b*)

Example 58. *There are infinite number of capacitors, each of capacitance 1 μF. They are connected in rows, such that the number of capacitors in the first row, second row, third row, fourth row, are respectively 1, 2, 4, 8, ... The rows of these capacitors are then connected between points A and B, as shown in Fig. 2.71. Determine the equivalent capacitance of the network between the points A and B.*

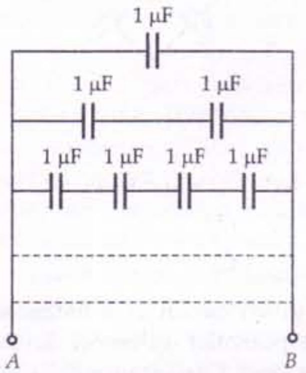


Fig. 2.71

Solution. Let $C_1, C_2, C_3, C_4, ...$ be the effective capacitances of the capacitors of first row, second row, third row, fourth row, ... respectively. Then

$$C_1 = 1\,\mu F$$

$$C_2 = \frac{1 \times 1}{1+1} = \frac{1}{2}\,\mu F$$

$$\frac{1}{C_3} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 4$$

$$\therefore \quad C_3 = \frac{1}{4}\,\mu F$$

Similarly, $C_4 = \frac{1}{8}\,\mu F$, and so on.

As these rows are connected in parallel between points *A* and *B*, so the equivalent capacitance between points *A* and *B* is

$$C = C_1 + C_2 + C_3 + C_4 + ... = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...$$

This is an infinite geometric progression with first term $a = 1$ and common ratio $r = 1/2$. Hence

$$C = \frac{a}{1-r} = \frac{1}{1 - 1/2} = \mathbf{2\,\mu F}.$$

Example 59. *Find the equivalent capacitor of the ladder (Fig. 2.72) between points A and B.*

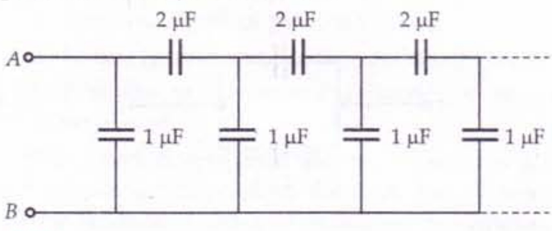


Fig. 2.72

Solution. Let *C* be the equivalent capacitance of the infinite network. It consists of repeating units of two capacitors of 1 μF and 2 μF. The addition of one such more unit will not affect the equivalent capacitance. But then the network would appear as shown in Fig. 2.73.

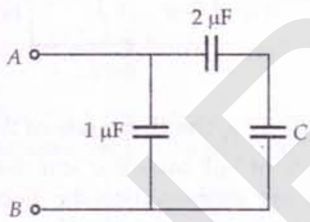


Fig. 2.73

The equivalent capacitance of the new arrangement must be equal to *C*.

$$\therefore \quad C = 1 + \frac{2 \times C}{2 + C}$$

or $\quad C^2 - C - 2 = 0$

or $\quad C = 2\,\mu F \;\text{ or }\; -1\,\mu F$

As the capacitance cannot be negative, so the equivalent capacitance of the ladder is **2 μF**.

Example 60. *If $C_1 = 20\,\mu F$, $C_2 = 30\,\mu F$ and $C_3 = 15\,\mu F$ and the insulated plate of C_1 be at a potential of 90 V, one plate of C_3 being earthed. What is the potential difference between the plates of C_2, three capacitors being connected in series ?* [CBSE OD 15]

Solution. Here $C_1 = 20\,\mu F$, $C_2 = 30\,\mu F$, $C_3 = 15\,\mu F$, $V = 90$ V

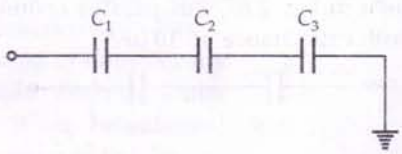


Fig. 2.74

The equivalent capacitance *C* of the series combination is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{20} + \frac{1}{30} + \frac{1}{15} = \frac{3}{20}$$

$$\therefore \quad C = \frac{20}{3}\,\mu F$$

Total potential difference $= 90 - 0 = 90$ V

∴ Total charge,

$$q = CV = \frac{20 \times 10^{-6}}{3} . 90 = 600 \times 10^{-6}\,C$$

P.D. between the plates of capacitor C_2 is

$$V_2 = \frac{q}{C_2} = \frac{600 \times 10^{-6}\,C}{30 \times 10^{-6}\,F} = \mathbf{20\,V}.$$

Example 61. *In the circuit shown in Fig. 2.75, if the point C is earthed and point A is given a potential of + 1200 V, find the charge on each capacitor and the potential at the point B.*

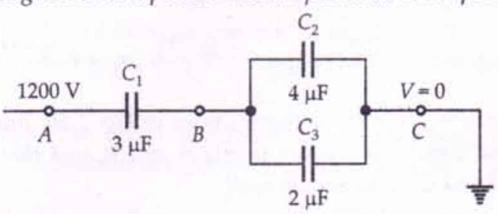


Fig. 2.75

Solution. Capacitors C_2 and C_3 form a parallel combination. Their equivalent capacitance is

$$C' = C_2 + C_3 = (4 + 2)\,\mu F = 6\,\mu F$$

Now C_1 and C' form a series combination, therefore, the equivalent capacitance of the entire network is

$$C = \frac{CC'}{C + C'} = \frac{3 \times 6}{3 + 6} = 2\,\mu F$$

The charge on the equivalent capacitor is

$$q = CV = 2 \times 10^{-6} \times 1200\,C = 2.4 \times 10^{-3}\,C$$

This must be equal to the charge on C_1 and also the sum of the charges on C_2 and C_3. Thus

$$V_A - V_B = \frac{q}{C_1} = \frac{2.4 \times 10^{-3}}{3 \times 10^{-6}} = 800\,V$$

$$V_A = 1200\,V$$

$$\therefore \quad V_B = 1200 - 800 = \mathbf{400\,V}$$

Hence $V_C - V_B = 400 - 0 = 400\,V$

$$q_2 = C_2(V_C - V_B) = 4 \times 10^{-6} \times 400\,C = \mathbf{1.6 \times 10^{-3}\,C}$$

$$q_3 = C_3(V_C - V_B) = 2 \times 10^{-6} \times 400\,C = \mathbf{0.8 \times 10^{-3}\,C}$$

$$q_1 = q = \mathbf{2.4 \times 10^{-3}\,C.}$$

Example 62. *A network of four 10 μF capacitors is connected to a 500 V supply as shown in Fig. 2.76. Determine (a) the equivalent capacitance of the network, (b) the charge on each capacitor.* [NCERT]

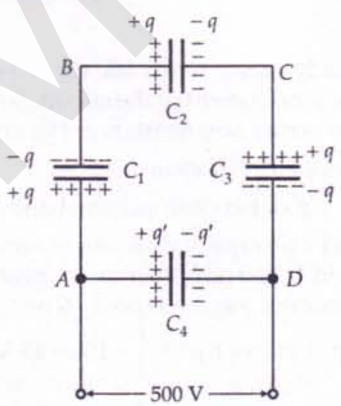


Fig. 2.76

Solution. (*a*) In the given network, C_1, C_2 and C_3 are connected in series. Their equivalent capacitance C' is given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10}$$

or $\quad C' = \frac{10}{3}\,\mu F$

Now C' and C_4 form a parallel combination. Therefore, the equivalent capacitance of the whole network is

$$C = C' + C_4 = \frac{10}{3} + 10 = \frac{40}{3}\,\mu F = \mathbf{13.3\,\mu F.}$$

(*b*) It is clear from Fig. 2.76 that the charge on each of the capacitors C_1, C_2 and C_3 is same. Let it be q. Let the charge on C_4 be q'.

$\therefore$ P.D. across AB, $\quad V_1 = \frac{q}{C_1}$

P.D. across BC, $\quad V_2 = \frac{q}{C_2}$

P.D. across CD, $\quad V_3 = \frac{q}{C_3}$

But $\quad V_1 + V_2 + V_3 = V$

$$\therefore \quad \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} = 500$$

or $$q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) = 500$$

or $$q \cdot \frac{1}{C'} = 500$$

$$\therefore \quad q = 500 \times C' = 500 \times \frac{10}{3}\,\mu C = \frac{5000}{3} \times 10^{-6}\,C$$

$$= \mathbf{1.7 \times 10^{-3}\,C}$$

Also, P.D. across AD $= \frac{q'}{C_4} = 500\,V$

$$\therefore \quad q' = 500 \times C_4 = 500 \times 10\,\mu C$$

$$= 5000 \times 10^{-6}\,C = \mathbf{5 \times 10^{-3}\,C.}$$

Example 63. *Four capacitors C_1, C_2, C_3 and C_4 are connected to a battery of 12 V, as shown in Fig. 2.77. Find the potential difference between the points A and B.*

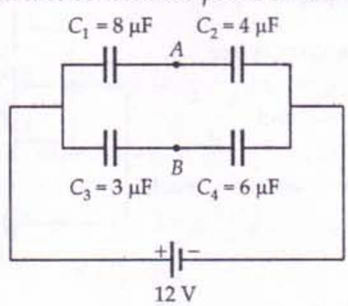


Fig. 2.77

Solution. Let V_A be the potential at point A and V_B that at B. Then

P.D. across $C_1 = 12 - V_A$

P.D. across $C_2 = V_A - 0 = V_A$

P.D. across $C_3 = 12 - V_B$

P.D. across $C_4 = V_B - 0 = V_B$

As the capacitors C_1 and C_2 are connected in series, so

$$q_1 = q_2$$

or $\quad C_1(12 - V_A) = C_2 V_A$

or $\quad 8(12 - V_A) = 4 V_A$

or $\quad V_A = 8\text{ V}$

Again, the capacitors C_3 and C_4 are connected in series, so

$$q_3 = q_4$$

or $\quad C_3(12 - V_B) = C_4 V_B$

or $\quad 3(12 - V_B) = 6 V_B$

or $\quad V_B = 4\text{ V}$

The potential difference between the points A and B is

$$V_A - V_B = 8 - 4 = \mathbf{4\ V}.$$

Example 64. *Five identical capacitor plates, each of area A are arranged such that the adjacent plates are at distance d apart. The plates are connected to a source of emf V, as shown in Fig. 2.78. Find the charges on the various plates.* [IIT 84]

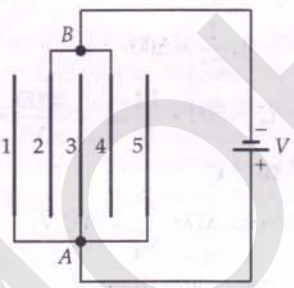


Fig. 2.78

Solution. As shown in Fig. 2.79, the given network is equivalent to three parallel-plate capacitors connected in parallel.

Their capacitances are

$$\frac{\varepsilon_0 A}{d}, \quad \frac{2\varepsilon_0 A}{d} \quad \text{and} \quad \frac{\varepsilon_0 A}{d}$$

The p.d. across each capacitor is V.

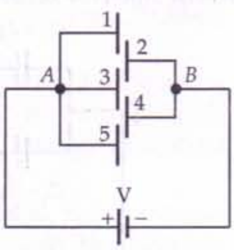


Fig. 2.79

As

Charge = Capacitance × p.d.

So charges on various plates are

$$q_1 = +\frac{\varepsilon_0 AV}{d}, \quad q_2 = -\frac{2\varepsilon_0 AV}{d}$$

$$q_3 = +\frac{2\varepsilon_0 AV}{d}, q_4 = -\frac{2\varepsilon_0 AV}{d}, q_5 = +\frac{\varepsilon_0 AV}{d}.$$

Example 65. *For the network shown in Fig. 2.80, find the potential difference between points A and B, and that between B and C in the steady state.*

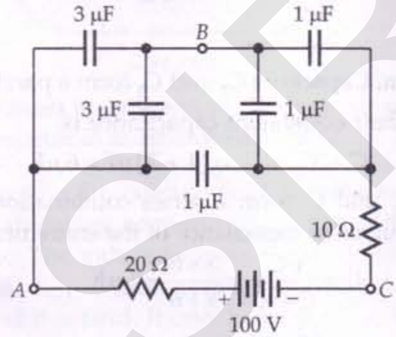


Fig. 2.80

Solution. The two capacitors of 3 μF and 3 μF on the left side of the network are in parallel, their equivalent capacitance = 6 μF

The two capacitors of 1 μF and 1 μF on the other side of the network are also in parallel, their equivalent capacitance = 2 μF. So the given network reduces to the equivalent circuit shown in Fig. 2.81.

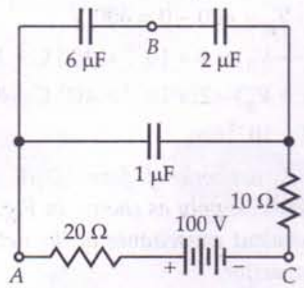


Fig. 2.81

In the steady state, when all the capacitors are charged, there is no current in the circuit. So there is no potential drop across any resistance. Hence

p.d. across 1 μF capacitor

= p.d. between points A and C = 100 V

As 6 μF and 2 μF capacitances are in series, the p.d. of 100 V is divided between them in the inverse ratio of their capacitances *i.e.*, in the ratio 2 : 6 or 1 : 3.

$$\therefore \quad V_{AB} = \text{p.d. across 6 μF} = \frac{1}{4} \times 100 = \mathbf{25\ V}$$

$$V_{BC} = \text{p.d. across 2 μF} = \frac{3}{4} \times 100 = \mathbf{75\ V}.$$

Problems For Practice

1. Two capacitors have a capacitance of 5 μF when connected in parallel and 1.2 μF when connected in series. Calculate their capacitances.
(Ans. 2 μF, 3 μF)

2. Two capacitors of equal capacitance when connected in series have net capacitance C_1, and when connected in parallel have net capacitance C_2. What is the value of C_1 / C_2 ? **[CBSE D 93C]**
(Ans. $C_1 / C_2 = 1/4$)

3. Three capacitors of capacity 1, 2 and 3 μF are connected such that second and third are in series and the first one in parallel. Calculate the resultant capacity. (Ans. 2.2 μF)

4. The capacities of three capacitors are in the ratio 1 : 2 : 3. Their equivalent capacity in parallel is greater than the equivalent capacity in series by 60/11 pF. Calculate the individual capacitances.
(Ans. 1 pF, 2 pF, 3 pF)

5. The equivalent capacitance of the combination between *A* and *B* in Fig. 2.82 is 4 μF.

Fig. 2.82

(*i*) Calculate capacitance of the capacitor *C*.
(*ii*) Calculate charge on each capacitor if a 12 V battery is connected across terminals *A* and *B*.
(*iii*) What will be the potential drop across each capacitor ? **[CBSE D 09]**
[Ans. (*i*) 5 μF (*ii*) 48 μC (*iii*) 2.4 V, 9.6 V]

6. How would you connect 8, 12 and 24 μF capacitors to obtain (*i*) minimum capacitance (*ii*) maximum capacitance ? If a potential difference of 100 volt is applied across the system, what would be the charges on the capacitors in each case ?
[Ans. (*i*) In series, $C_{min} = 4\ \mu F$, $q = 4\ \mu C$,
(*ii*) In parallel, $C_{max} = 44\ \mu F$, $q_1 = 800\ \mu C$,
$q_2 = 1200\ \mu C$, $q_3 = 2400\ \mu C$]

7. Calculate the capacitance of the capacitor in Fig. 2.83, if the equivalent capacitance of the combination between *A* and *B* is 15 μF. **[CBSE D 94]**
(Ans. 60 μF)

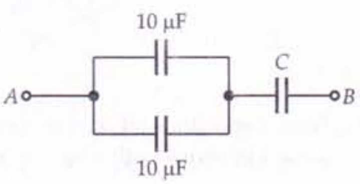


Fig. 2.83

8. In the combination of four identical capacitors shown in Fig. 2.84, the equivalent capacitance between points *P* and *Q* is 1 μF. Find the value of each separate capacitance. (Ans. 4 μF)

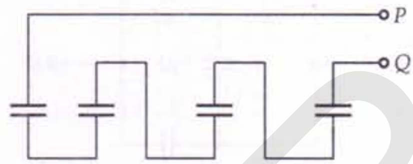


Fig. 2.84

9. Find the equivalent capacitance of the combination shown in Fig. 2.85 between the points *A* and *B*.
$\left(\text{Ans. } C_3 + \dfrac{2C_1C_2}{C_1 + C_2}\right)$

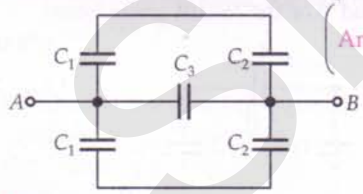


Fig. 2.85

10. For the network shown in Fig. 2.86, calculate the equivalent capacitance between points *A* and *B*.
(Ans. 6 μF)

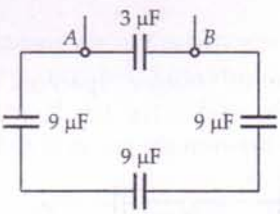


Fig. 2.86

11. Calculate the capacitance of the capacitor *C* in Fig. 2.87. The equivalent capacitance of the combination between *P* and *Q* is 30 μF. **[CBSE OD 95]**
(Ans. 60 μF)

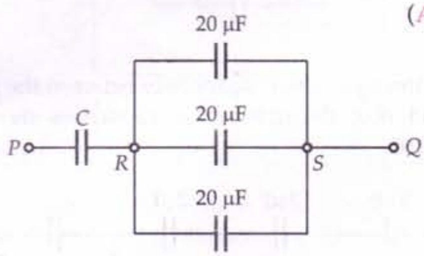


Fig. 2.87

12. Calculate the equivalent capacitance between points *A* and *B* of the combination shown in Fig. 2.88.
(Ans. 0.5 μF)

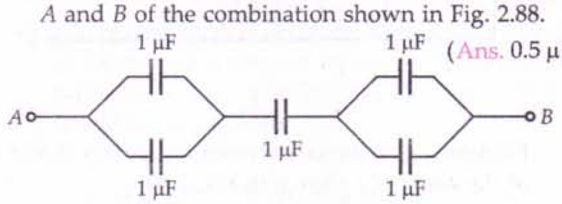


Fig. 2.88

13. Find the equivalent capacitance between points A and B for the network shown in Fig. 2.89. $\left(\text{Ans. } \frac{8}{3}\,\mu F\right)$

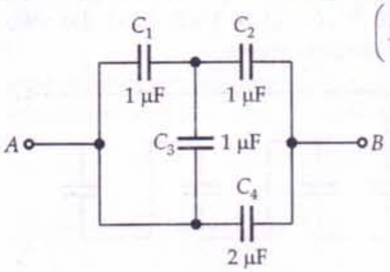


Fig. 2.89

14. Calculate the equivalent capacitance between the points A and B of the circuit given below. **[CBSE F 95]** $\left(\text{Ans. } \frac{28}{19}\,\mu F\right)$

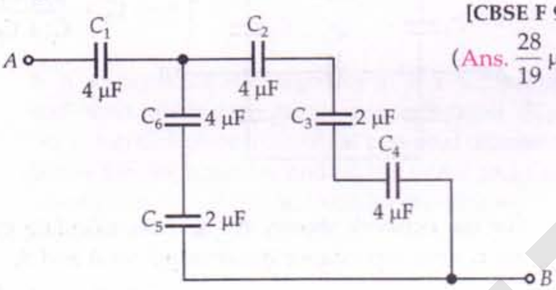


Fig. 2.90

15. A network of six identical capacitors, each of value C is made, as shown in Fig. 2.91. Find the equivalent capacitance between the points A and B. (Ans. $4C/3$)

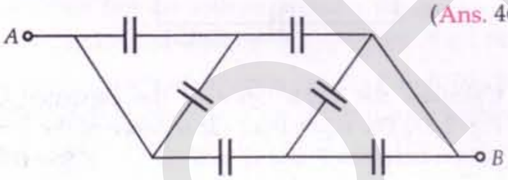


Fig. 2.91

16. Find the equivalent capacitance between the points A and B of the network of capacitors shown in Fig. 2.92. (Ans. $1\,\mu F$)

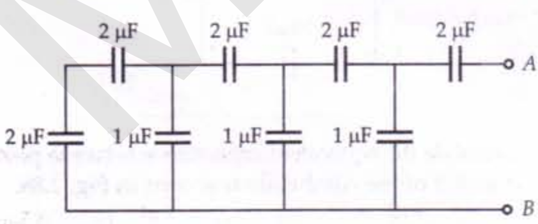


Fig. 2.92

17. Find the capacitance between the points A and B of the assembly shown in Fig. 2.93. (Ans. $2.25\,\mu F$)

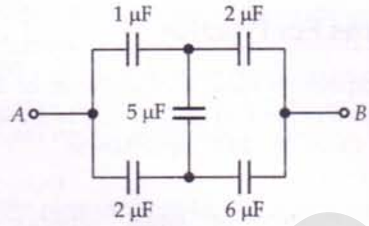


Fig. 2.93

18. Find the resultant capacitance between the points X and Y of the combination of capacitors shown in Fig. 2.94. **[Haryana 01]** (Ans. $2.5\,\mu F$)

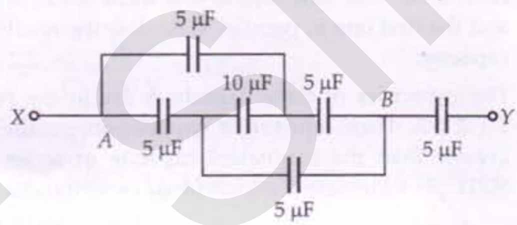


Fig. 2.94

19. The outer cylinders of two cylindrical capacitors of capacitance $2.2\,\mu F$ each are kept in contact and the inner cylinders are connected through a wire. A battery of emf 10 V is connected, as shown in Fig. 2.95. Find the charge supplied by the battery to the inner cylinders. (Ans. $44\,\mu C$)

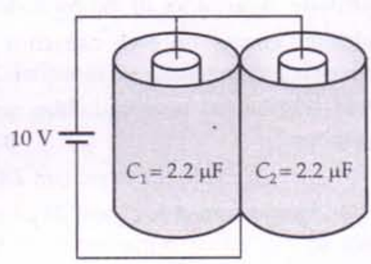


Fig. 2.95

20. In Fig. 2.96, $C_1 = 1\,\mu F$, $C_2 = 2\,\mu F$ and $C_3 = 3\,\mu F$. Find the equivalent capacitance between points A and B. (Ans. $6\,\mu F$)

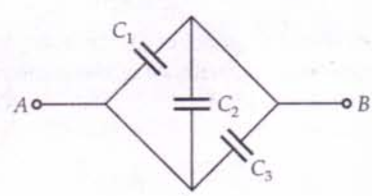


Fig. 2.96

21. Four capacitors of equal capacitances are connected in series with a battery of 10 V, as shown in

Fig. 2.97. The middle point B is connected to the earth. What will be the potentials of the points A and C ? (Ans. $V_A = +5\text{ V}, V_C = -5\text{ V}$)

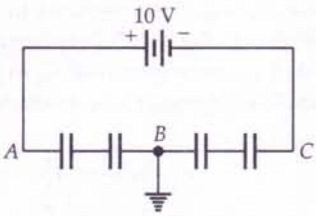


Fig. 2.97

22. Determine the potential difference across the plates of each capacitor of the network shown in Fig. 2.98. Take $E_2 > E_1$.

$$\left(\text{Ans. } V_1 = \frac{(E_2 - E_1)\,C_2}{C_1 + C_2},\; V_2 = \frac{(E_2 - E_1)\,C_1}{C_1 + C_2}\right)$$

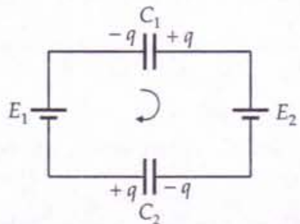


Fig. 2.98

23. Find the potential difference between the points A and B of the arrangement shown in Fig. 2.99. (Ans. – 8 V)

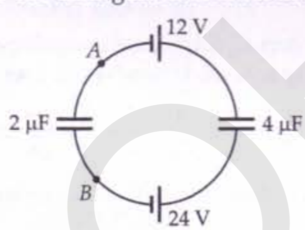


Fig. 2.99

24. Determine the potential difference $V_A - V_B$ between points A and B of the circuit shown in Fig. 2.100. Under what condition is it equal to zero ?

$$\left[\text{Ans. } V\left(\frac{C_2 C_3 - C_1 C_4}{(C_1 + C_2)(C_3 + C_4)}\right),\; \frac{C_1}{C_2} = \frac{C_3}{C_4}\right]$$

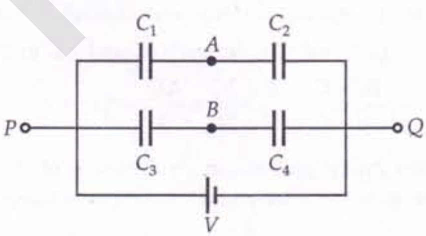


Fig. 2.100

25. A variable capacitor has n plates and the distance between two successive plates is d. Determine its capacitance.

$$\left(\text{Ans. } C = \frac{(n-1)\,\varepsilon_0 A}{d}\right)$$

26. A network of four capacitors each of 12 µF capacitance is connected to a 500 V supply as shown in Fig. 2.101. Determine

(*a*) equivalent capacitance of the network, and

(*b*) charge on each capacitor. **[CBSE OD 10]**

[Ans. (*a*) 16 µF (*b*) $q_1 = q_2 = q_3 = 2000\ \mu\text{C}$, $q_4 = 6000\ \mu\text{C}$]

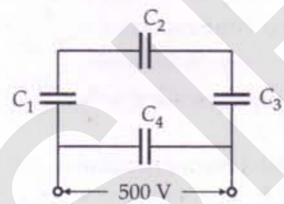


Fig. 2.101

27. For the network shown in Fig. 2.102, compute

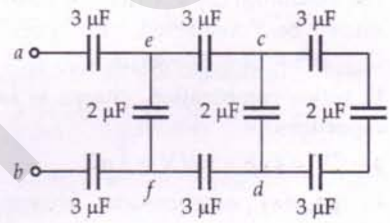


Fig. 2.102

(*i*) the equivalent capacitance between points a and b.

(*ii*) the charge on each of the capacitors nearest to a and b when $V_{ab} = 900$ V.

(*iii*) V_{cd}, when $V_{ab} = 900$ V.

[Ans. (*i*) 1 µF (*ii*) 900 µC (*iii*) 100 V]

HINTS

1. Proceed as in Example 45 on page 2.35.
2. Proceed as in Example 46 on page 2.35.
3. $C = 1 + \frac{2 \times 3}{2 + 3} = \mathbf{2.2\ \mu F}$.
4. Let the capacitances be C, $2C$ and $3C$. Then

$$C_p = C + 2C + 3C = 6C$$

$$\frac{1}{C_s} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{3C} = \frac{11}{6C} \quad \text{or} \quad C_s = \frac{6C}{11}$$

Given $C_p - C_s = \frac{60}{11}$ pF or $6C - \frac{6C}{11} = \frac{60}{11}$ pF

or $C = 1$ pF

So the individual capacitances are 1 pF, 2 pF and 3 pF.

5. (*i*) As 20 μF capacitor and capacitor C are in series, their equivalent capacitance is

$$C_{AB}=\frac{C\times 20}{C+20}$$

or $$4\,\mu\text{F}=\frac{20C}{C+20}$$

or $$4C+80=20C$$

or $$C=\mathbf{5\,\mu F}.$$

(*ii*) Charge on each capacitor,

$$q=C_{AB}V=4\,\mu\text{F}\times 12\,\text{V}=\mathbf{48\,\mu C}.$$

(*iii*) P.D. on 20 μF capacitor $=\frac{q}{20\,\mu\text{F}}=\frac{48\,\mu\text{C}}{20\,\mu\text{F}}=\mathbf{2.4\ V}$

P.D. on capacitor $C=\frac{q}{C}=\frac{48\,\mu\text{C}}{5\,\mu\text{F}}=\mathbf{9.6\ V}.$

6. (*i*) For minimum capacitance, the three capacitors must be connected in series. Then

$$\frac{1}{C_{min}}=\frac{1}{8}+\frac{1}{12}+\frac{1}{24}=\frac{1}{4}\quad\text{or}\quad C_{min}=\mathbf{4\,\mu F}.$$

(*ii*) For maximum capacitance, the three capacitors must be connected in parallel. Then $C_{max}=8+12+24=\mathbf{44\,\mu F}.$

(*iii*) In series combination, charge is same on all capacitors.

$$q=CV=4\,\mu\text{F}\times 100\ \text{V}=4\,\mu\text{C}.$$

In parallel combination, charges on the capacitors are

$$q_1=C_1V=8\,\mu\text{F}\times 100\ \text{V}=\mathbf{800\,\mu C}$$
$$q_2=C_2V=12\,\mu\text{F}\times 100\ \text{V}=\mathbf{1200\,\mu C}$$
$$q_3=C_3V=24\,\mu\text{F}\times 100\ \text{V}=\mathbf{2400\,\mu C}.$$

7. The combined capacitance of the parallel combination of two 10 μF capacitors is 20 μF. This combination is connected in series with capacitance C.

$$\therefore\quad \frac{1}{20}+\frac{1}{C}=\frac{1}{15}\quad\text{or}\quad \frac{1}{C}=\frac{1}{15}-\frac{1}{20}=\frac{4-3}{60}=\frac{1}{60}$$

or $$C=\mathbf{60\,\mu F}.$$

8. All capacitors are in series.

$$\therefore\quad \frac{4}{C}=\frac{1}{1\,\mu\text{F}}\quad\text{or}\quad C=\mathbf{4\,\mu F}.$$

9. $$C=\frac{C_1C_2}{C_1+C_2}+C_3+\frac{C_1C_2}{C_1+C_2}=C_3+\frac{2C_1C_2}{C_1+C_2}.$$

10. $$\frac{1}{C'}=\frac{1}{9}+\frac{1}{9}+\frac{1}{9}=\frac{1}{3},\quad C'=3\,\mu\text{F}$$

$$C=3\,\mu\text{F}+C'=3\,\mu\text{F}+3\,\mu\text{F}=\mathbf{6\,\mu F}.$$

11. $$\frac{1}{C}+\frac{1}{60}=\frac{1}{30}\quad\therefore\quad C=\mathbf{60\,\mu F}.$$

12. $$\frac{1}{C}=\frac{1}{1+1}+\frac{1}{1}+\frac{1}{1+1}=\frac{2}{1}\quad\therefore\quad C=\mathbf{0.5\,\mu F}.$$

13. C_1 and C_3 are in parallel between points A and D. So the equivalent capacitance between A and D is

$$C'=C_1+C_3=1+1=2\,\mu\text{F}$$

The given network now reduces to the equivalent circuit shown in Fig. 2.103. Between points A and B, now C' and C_2 are in series and C_4 in parallel. Hence the equivalent capacitance between A and B is

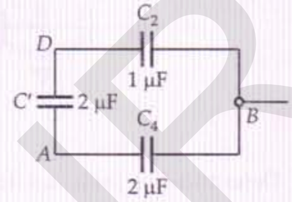


Fig. 2.103

$$C=\frac{C'C_2}{C'+C_2}+C_4=\frac{2\times 1}{2+1}+2=\frac{8}{3}\,\mu\text{F}.$$

14. Capacitors C_2, C_3 and C_4 are connected in series, their equivalent capacitance C_7 is given by

$$\frac{1}{C_7}=\frac{1}{C_2}+\frac{1}{C_3}+\frac{1}{C_4}=\frac{1}{4}+\frac{1}{2}+\frac{1}{4}=\frac{1}{1}$$

$$\therefore\quad C_7=1\,\mu\text{F}$$

Also, C_5 and C_6 are in series, the equivalent capacitance is

$$C_8=\frac{C_5\times C_6}{C_5+C_6}=\frac{2\times 4}{2+4}=\frac{4}{3}\,\mu\text{F}$$

C_7 and C_8 form a parallel combination of capacitance,

$$C_9=C_7+C_8=1+\frac{4}{3}=\frac{7}{3}\,\mu\text{F}$$

Now C_1 and C_9 form a series combination. The equivalent capacitance C between A and B is given by

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_9}=\frac{1}{4}+\frac{3}{7}=\frac{19}{28}\quad\text{or}\quad C=\mathbf{\frac{28}{19}}\,\mu\text{F}.$$

15. The equivalent network is shown in Fig. 2.104.

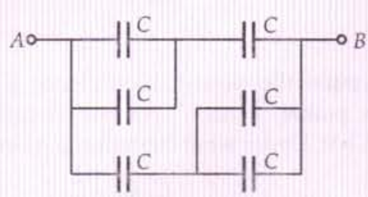


Fig. 2.104

Clearly, the equivalent capacitance

$$=[2\,C\text{ and }C\text{ in series}]\,||\,[C\text{ and }2\,C\text{ in series}]$$

$$=\frac{2C\times C}{2C+C}+\frac{C\times 2C}{C+2C}=\mathbf{\frac{4C}{3}}.$$

16. Two 2 μF capacitors at the left side of the network are in series. Their equivalent capacitance is

$$C_s=\frac{2\times 2}{2+2}=1\,\mu\text{F}$$

The capacitance C_s and the next capacitor of $1\,\mu F$ are in parallel. Their equivalent capacitance is

$$C_p = 1 + 1 = 2\,\mu F$$

Proceeding in this way, we finally get two $2\,\mu F$ capacitors connected in series.

$\therefore$ Equivalent capacitance between A and B

$$= \frac{2 \times 2}{2 + 2} = \mathbf{1\,\mu F}.$$

17. The given arrangement is a balanced wheatstone bridge. Proceed as in Example 57 on page 2.39.

18. The arrangement between the points A and B is a balanced wheatstone bridge. Proceeding as in Example 57, we find that the equivalent capacitance between A and B is

$$C' = 5\,\mu F$$

Now the capacitor C' and the left out capacitor of $5\,\mu F$ are in series. The equivalent capacitance between points X and Y will be

$$C = \frac{C' \times 5}{C' + 5} = \frac{5 \times 5}{5 + 5} = \mathbf{2.5\,\mu F}.$$

19. The two capacitors are connected in parallel

$\therefore$ $C = 2.2 + 2.2 = 4.4\,\mu F$

Charge, $q = CV = 4.4\,\mu F \times 10\text{ V} = \mathbf{44\,\mu C}$.

20. The three capacitors are connected in parallel between points A and B.

$\therefore$ $C = C_1 + C_2 + C_3 = 1 + 2 + 3 = \mathbf{6\,\mu F}$.

21. Here $V_B = 0$. As the capacitances are equal on the two sides of point B,

$\therefore$ $V_A - V_B = V_B - V_C$

or $V_A + V_C = 2V_B = 0$

But $V_A - V_C = 10\text{ V}$

$\therefore$ $V_A = \mathbf{+5\text{ V}}$ and $V_C = \mathbf{-5\text{ V}}$.

22. Let charge q flow across the capacitor plates until the current stops. In a closed circuit,

$$\Sigma \Delta V = 0$$

or $$E_1 + \frac{q}{C_1} - E_2 + \frac{q}{C_2} = 0$$

or $$q\left(\frac{C_1 + C_2}{C_1 C_2}\right) = E_2 - E_1$$

or $$q = \frac{(E_2 - E_1)\, C_1 C_2}{C_1 + C_2}$$

P.D. across plates of $C_1 = \dfrac{q}{C_1} = \dfrac{(E_2 - E_1)\, C_2}{C_1 + C_2}$

P.D. across plates of $C_2 = \dfrac{q}{C_2} = \dfrac{(E_2 - E_1)\, C_1}{C_1 + C_2}$.

23. The given arrangement is equivalent to the circuit shown in Fig. 2.105.

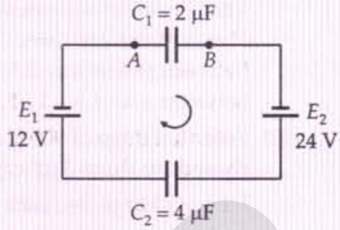


Fig. 2.105

Proceeding as in the above problem 22, we get

$$q = \frac{(E_2 - E_1)\, C_1 C_2}{C_1 + C_2}$$

P.D. across the plates of C_1,

$$V_1 = \frac{q}{C_1} = \frac{(E_2 - E_1)\, C_2}{C_1 + C_2} = \frac{(12\text{ V} - 24\text{ V})\, 4\,\mu F}{2\,\mu F + 4\,\mu F} = \mathbf{-8\text{ V}}.$$

24. Suppose the charge q_1 flows in the upper branch and q_2 in the lower branch. Then

$$V = q_1\left(\frac{1}{C_1} + \frac{1}{C_2}\right) \quad \text{or} \quad q_1 = \frac{VC_1C_2}{C_1 + C_2}$$

Also, $$V = q_2\left(\frac{1}{C_3} + \frac{1}{C_4}\right)$$

or $$q_2 = \frac{VC_3C_4}{C_3 + C_4}$$

$$\therefore V_A - V_B = (V_Q - V_B) - (V_Q - V_A) = \frac{q_2}{C_4} - \frac{q_1}{C_2}$$

Putting the values of q_1 and q_2, we get

$$V_A - V_B = \frac{VC_3}{C_3 + C_4} - \frac{VC_1}{C_1 + C_2}$$

$$= V\left[\frac{C_2C_3 - C_1C_4}{(C_1 + C_2)(C_3 + C_4)}\right]$$

For $V_A - V_B = 0$, we have

$$C_2C_3 - C_1C_4 = 0 \quad \text{or} \quad \frac{C_1}{C_2} = \frac{C_3}{C_4}.$$

25. The given arrangement is equivalent to $(n-1)$ capacitors joined in parallel.

$$\therefore \quad C = \frac{(n-1)\,\varepsilon_0 A}{d}.$$

26. (*a*) $C_{123} = \dfrac{12\,\mu F}{3} = 4\,\mu F$

$C_{eq} = C_{123} + C_4 = 4 + 12 = \mathbf{16\,\mu F}$.

(*b*) $q_1 = q_2 = q_3 = C_{123} V = 4\,\mu F \times 500\text{V} = \mathbf{2000\,\mu C}$

$q_4 = C_4 V = 12\,\mu F \times 500\text{ V} = \mathbf{6000\,\mu C}$.

27. (*i*) Three $3\,\mu F$ capacitors in series have equivalent capacitance $= 1\,\mu F$. The combination is in parallel with $2\,\mu F$ capacitor.

$\therefore$ Equivalent capacitance between c and d

$$= 1 + 2 = 3\,\mu F$$

The situation is repeated for points e and f. Hence there are three $3\,\mu F$ capacitors in series between points a and b. Equivalent capacitance between a and $b = \mathbf{1\,\mu F}$.

(*ii*) Potential drop of 900 V across a and b is equally shared by three $3\,\mu F$ capacitors.

Hence charge on each capacitor nearest to a and b

$$= 300 \times 3 = \mathbf{900\,\mu C}$$

(*iii*) Potential drop of 300 V across e and f is equally shared by $3\,\mu F$ capacitors.

Hence $\quad V_{cd} = \mathbf{100\ V.}$

2.24 ENERGY STORED IN A CAPACITOR

38. *How does a capacitor store energy ? Derive an expression for the energy stored in a capacitor.*

Energy stored in a capacitor. A capacitor is a device to store energy. The process of charging up a capacitor involves the transferring of electric charges from its one plate to another. The *work done in charging the capacitor is stored as its electrical potential energy.* This energy is supplied by the battery at the expense of its stored chemical energy and can be recovered by allowing the capacitor to discharge.

Expression for the energy stored in a capacitor. Consider a capacitor of capacitance C. Initially, its two plates are uncharged. Suppose the positive charge is transferred from plate 2 to plate 1 bit by bit. In this process, external work has to be done because at any stage plate 1 is at higher potential than the plate 2. Suppose at any instant the plates 1 and 2 have charges Q' and $-Q'$ respectively, as shown in Fig. 2.106(*a*). Then the potential difference between the two plates will be

$$V' = \frac{Q'}{C}$$

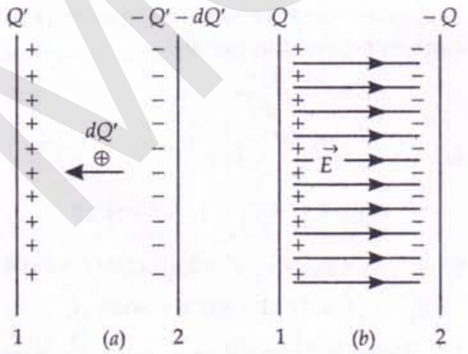


Fig. 2.106 (*a*) Work done in transferring charge dQ' from plate 2 to plate 1. (*b*) Total work done in charging the capacitor may be considered as the energy stored in the electric field between the plates.

Suppose now a small additional charge dQ' be transferred from plate 2 to plate 1. The work done will be

$$dW = V'.dQ' = \frac{Q'}{C}.dQ'$$

The total work done in transferring a charge Q from plate 2 to plate 1 [Fig. 2.105(*b*)] will be

$$W = \int dW = \int_0^Q \frac{Q'}{C}.dQ' = \left[\frac{Q'^2}{2C}\right]_0^Q = \frac{1}{2}.\frac{Q^2}{C}$$

This work done is stored as electrical potential energy U of the capacitor.

$$U = \frac{1}{2}.\frac{Q^2}{C} = \frac{1}{2}.CV^2 = \frac{1}{2}QV \qquad [\because Q = CV]$$

39. *If several capacitors are connected in series or parallel, show that the energy stored would be additive in either case.*

Energy stored in a series combination of capacitors.

For a series combination, $\quad Q = \text{constant}$

Total energy,

$$U = \frac{Q^2}{2}.\frac{1}{C} = \frac{Q^2}{2}.\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...\right]$$

$$= \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3} + ...$$

or $\quad U = U_1 + U_2 + U_3 + ...$

Energy stored in a parallel combination of capacitors. For a parallel combination, $V = \text{constant}$

Total energy,

$$U = \frac{1}{2}CV^2 = \frac{1}{2}[C_1 + C_2 + C_3 + ...]V^2$$

$$= \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \frac{1}{2}C_3V^2 + ...$$

or $\quad U = U_1 + U_2 + U_3 + ...$

Hence *total energy is additive both in series and parallel combinations of capacitors.*

2.25 ENERGY DENSITY OF AN ELECTRIC FIELD

40. *Where is the energy stored in a capacitor ? Derive an expression for the energy density of an electric field.*

Energy density of an electric field. When a capacitor is charged, an electric field is set up in the region between its two plates. We can say that the work done in the charging process has been used in creating the electric field. Thus the presence of an electric field implies stored energy or *the energy is stored in the electric field.*

Consider a parallel plate capacitor, having plate area A and plate separation d. Capacitance of the parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d}$$

If σ is the surface charge density on the capacitor plates, then electric field between the capacitor plates will be

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{or} \quad \sigma = \varepsilon_0 E$$

Charge on either plate of capacitor is

$$Q = \sigma A = \varepsilon_0 EA$$

$\therefore$ Energy stored in the capacitor is

$$U = \frac{Q^2}{2C} = \frac{(\varepsilon_0 EA)^2}{2 \cdot \frac{\varepsilon_0 A}{d}} = \frac{1}{2} \varepsilon_0 E^2 Ad$$

But Ad = volume of the capacitor between its two plates. Therefore, the *energy stored per unit volume or the energy density* of the electric field is given by

$$u = \frac{U}{Ad} = \frac{1}{2} \varepsilon_0 E^2$$

Although we have derived the above equation for a parallel plate capacitor, it is true for electric field due to any charge configuration. In general, we can say that *an electric field E can be regarded as a seat of energy with energy density equal to* $\frac{1}{2} \varepsilon_0 E^2$. Similarly, energy is also associated with a magnetic field.

2.26 REDISTRIBUTION OF CHARGES

41. *If two charged conductors are touched mutually and then separated, prove that the charges on them will be divided in the ratio of their capacitances.*

Redistribution of charges. Consider two insulated conductors A and B of capacitances C_1 and C_2, and carrying charges Q_1 and Q_2 respectively. Let V_1 and V_2 be their respective potentials. Then

$$Q_1 = C_1 V_1 \quad \text{and} \quad Q_2 = C_2 V_2$$

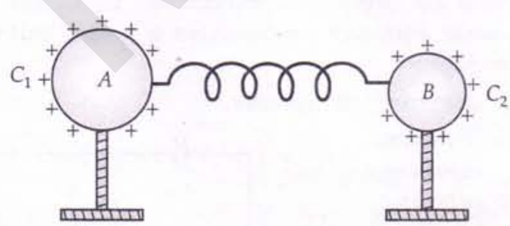


Fig. 2.107 Redistribution of charges.

Now, if the two conductors are joined by a thin conducting wire, then the positive charge will flow from the conductor at higher potential to that at lower potential till their potentials become equal. Thus the charges are redistributed. But the total charge still remains $Q_1 + Q_2$.

If the capacitance of the thin connecting wire is negligible and the conductors are a sufficient distance apart so that do not exert mutual electric forces, then their combined capacitance will be $C_1 + C_2$.

$$\text{Common potential} = \frac{\text{Total charge}}{\text{Total capacitance}}$$

or
$$V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

If after redistribution charges on A and B are Q_1' and Q_2' respectively, then

$$Q_1' = C_1 V \quad \text{and} \quad Q_2' = C_2 V$$

$$\therefore \quad \frac{Q_1'}{Q_2'} = \frac{C_1}{C_2}$$

Thus, after redistribution, the charges on the two conductors are in the ratio of their capacitances.

42. *When two charged conductors having different capacities and different potentials are joined together, show that there is always a loss of energy.*

Loss of energy in redistribution of charges. Let C_1 and C_2 be the capacitances and V_1 and V_2 be the potentials of the two conductors before they are connected together. Potential energy before connection is

$$U_i = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$$

After connection, let V be their common potential. Then

$$V = \frac{\text{Total charge}}{\text{Total capacitance}} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

Potential energy after connection is

$$U_f = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 = \frac{1}{2}(C_1 + C_2) V^2$$

$$= \frac{1}{2}(C_1 + C_2)\left[\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}\right]^2$$

$$= \frac{1}{2} \cdot \frac{(C_1 V_1 + C_2 V_2)^2}{(C_1 + C_2)}$$

Loss in energy,

$$U = U_i - U_f$$

$$= \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 - \frac{1}{2} \cdot \frac{(C_1 V_1 + C_2 V_2)^2}{(C_1 + C_2)}$$

$$= \frac{1}{2(C_1 + C_2)} [C_1^2 V_1^2 + C_1 C_2 V_1^2 + C_1 C_2 V_2^2 + C_2^2 V_2^2 - C_1^2 V_1^2 - C_2^2 V_2^2 - 2 C_1 C_2 V_1 V_2]$$

$$= \frac{1}{2} \frac{C_1 C_2}{(C_1 + C_2)} [V_1^2 + V_2^2 - 2 V_1 V_2]$$

$$= \frac{1}{2} \cdot \frac{C_1 C_2 (V_1 - V_2)^2}{C_1 + C_2}$$

This is always positive whether $V_1 > V_2$ or $V_1 < V_2$. So when two charged conductors are connected, charges flow from higher potential side to lower potential side till the potentials of the two conductors get equalised. In doing so, there is always some loss of potential energy in the form of heat due to the flow of charges in connecting wires.

Examples based on Energy Stored in Capacitors

Formulae Used

1. Energy stored in a capacitor,
$$U = \frac{1}{2} CV^2 = \frac{1}{2} \cdot \frac{q^2}{C} = \frac{1}{2} qV$$
2. Energy stored per unit volume or the energy density of the electric field of a capacitor,
$$u = \frac{1}{2} \varepsilon_0 E^2$$
3. Electric field between capacitor plates, $E = \frac{\sigma}{\varepsilon_0}$

Units Used

Capacitance is in farad, charge in coulomb, electric field in NC^{-1} or Vm^{-1}, energy in joule and energy density in Jm^{-3}.

Example 66. *How much work must be done to charge a 24 μF capacitor when the potential difference between the plates is 500 V ?* [Haryana 02]

Solution. Here $C = 24\ \mu F = 24 \times 10^{-6}$ F, $V = 500$ V

Work done,
$$W = \frac{1}{2} CV^2 = \frac{1}{2} \times 24 \times 10^{-6} \times (500)^2 = \mathbf{3\ J}.$$

Example 67. *A capacitor is charged through a potential difference of 200 V, when 0.1 C charge is stored in it. How much energy will it release, when it is discharged ?* [ISCE 98]

Solution. Here $V = 200$ V, $q = 0.1$ C

Energy stored, $U = \frac{1}{2} qV = \frac{1}{2} \times 0.1 \times 200 = 10$ J

When the capacitor is discharged, it releases the same amount of energy *i.e.*, **10 J**.

Example 68. *Two parallel plates, separated by 2 mm of air, have a capacitance of 3×10^{-14} F and are charged to a potential of 200 V. Then without touching the plates, they are moved apart till the separation is 6 mm. (i) What is the potential difference between the plates ? (ii) What is the change in energy ?*

Solution. Charge, $q = CV = 3 \times 10^{-14} \times 200 = 6 \times 10^{-12}$ C

When the separation increases from 2 mm to 6 mm, the capacitance becomes

$$C' = \frac{d}{d'} \cdot C = \frac{2}{6} \times 3 \times 10^{-14} = 10^{-14}\ F$$

(*i*) P.D. between the plates becomes

$$V' = \frac{q}{C'} = \frac{6 \times 10^{-12}}{10^{-14}} = \mathbf{600\ V}.$$

(*ii*) Initial energy stored in the capacitor,

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times 3 \times 10^{-14} \times (200)^2 = 6 \times 10^{-10}\ J$$

Final energy stored in the capacitor

$$U' = \frac{1}{2} C'V'^2 = \frac{1}{2} \times 10^{-14} \times (600)^2 = 18 \times 10^{-10}\ J$$

Increase in energy $= U' - U = \mathbf{12 \times 10^{-10}\ J}$.

Example 69. *Two capacitors of capacitances $C_1 = 3\ \mu F$ and $C_2 = 6\ \mu F$ arranged in series are connected in parallel with a third capacitor $C_3 = 4\ \mu F$. The arrangement is connected to a 6.0 V battery. Calculate the total energy stored in the capacitors.* [CBSE Sample Paper 98]

Solution. Equivalent capacitance of the series combination of C_1 and C_2 is given by

$$C' = \frac{C_1 C_2}{C_1 + C_2} = \frac{3 \times 6}{3 + 6} = 2\ \mu F$$

Combination C' is in parallel with C_3.

∴ Total capacitance,

$$C = C' + C_3 = 2 + 4 = 6\ \mu F = 6 \times 10^{-6}\ F$$

Energy stored,

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times 6 \times 10^{-6} \times 6^2 = \mathbf{1.08 \times 10^{-4}\ J}.$$

Example 70. *Three identical capacitors C_1, C_2 and C_3 of capacitance 6 μF each are connected to a 12 V battery as shown. Find :*

(*i*) *charge on each capacitor.*

(*ii*) *equivalent capacitance of the network.*

(*iii*) *energy stored in the network of capacitors.*

[CBSE D09]

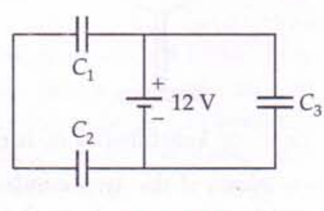


Fig. 2.108

Solution. (*i*) C_1 and C_2 are connected in series across 12 battery while C_3 is in parallel with this combination.

Equivalent capacitance of C_1 and C_2 is

$$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{6 \times 6}{6+6} = 3\ \mu\text{F}$$

Charge on either of the capacitors C_1 and C_2 is same.

$$q_1 = q_2 = C_{12} V = 3\ \mu\text{F} \times 12\ \text{V} = \mathbf{36\ \mu C}$$

Charge on C_3,

$$q_3 = 6 \mu\text{F} \times 12\ \text{V} = \mathbf{72\ \mu C}$$

(*ii*) Equivalent capacitance of the network,

$$C = C_{12} + C_3 = 3\ \mu\text{F} + 6\ \mu\text{F} = \mathbf{9\ \mu F}.$$

(*iii*) Energy stored in the network,

$$U = \frac{1}{2} C V^2 = \frac{1}{2} \times 9 \times 10^{-6} \times (12)^2 = \mathbf{6.48 \times 10^{-4}\ J}.$$

Example 71. *In Fig. 2.109, the energy stored in C_4 is 27 J. Calculate the total energy stored in the system.*

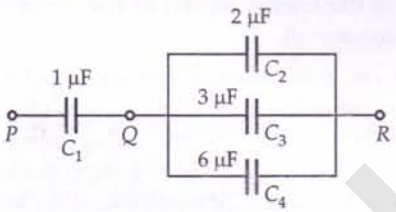


Fig. 2.109

Solution. Energy stored in C_4 is

$$U_4 = \frac{1}{2} C_4 V^2 = 27\ \text{J}$$

or $$\frac{1}{2} \times 6 \times 10^{-6} \times V^2 = 27$$

or $$V^2 = \frac{27 \times 2}{6 \times 10^{-6}} = 9 \times 10^6$$

Energy stored in C_2,

$$U_2 = \frac{1}{2} \times 2 \times 10^{-6} \times 9 \times 10^6 = 9\ \text{J}$$

Energy stored in C_3,

$$U_3 = \frac{1}{2} \times 3 \times 10^{-6} \times 9 \times 10^6 = 13.5\ \text{J}$$

Energy stored in C_2, C_3 and C_4

$$= U_2 + U_3 + U_4 = 9 + 13.5 + 27 = 49.5\ \text{J}$$

Equivalent capacitance of C_2, C_3 and C_4 connected in parallel

$$= 2 + 3 + 5 = 11\ \mu\text{F}$$

$$\therefore \quad \frac{q^2}{2 \times 11 \times 10^{-6}} = 49.5\ \text{J} \qquad \left[U = \frac{q^2}{2C}\right]$$

Energy stored in C_1,

$$U_1 = \frac{q^2}{2C_1} = \frac{49.5 \times 2 \times 11 \times 10^{-6}}{2 \times 1 \times 10^{-6}} = 544.5\ \text{J}$$

Total energy stored in the arrangement

$$= 544.5 + 49.5 = \mathbf{594.0\ J}.$$

Example 72. *In a camera-flash circuit (Fig. 2.110), a 2000 μF capacitor is charged by a 1.5 V cell. When a flash is required, the energy stored in the capacitor is discharged by means of a trigger T through a discharge tube in 0.1 millisecond. Find the energy stored in the capacitor and the power of the flash.* [ISCE 97]

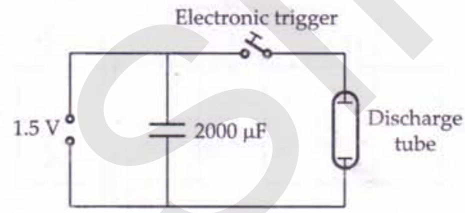


Fig. 2.110

Solution. Here $C = 2000\ \mu\text{F} = 2 \times 10^{-3}$ F, $V = 1.5$ V

Energy stored in the capacitor,

$$U = \frac{1}{2} C V^2 = \frac{1}{2} \times 2 \times 10^{-3} \times (1.5)^2 = 2.25 \times 10^{-3}\ \text{J}$$

Time during which capacitor is discharged for producing flash,

$$t = 0.1\ \text{millisecond} = 0.1 \times 10^{-3}\ \text{s} = 10^{-4}\ \text{s}$$

Power of flash, $$P = \frac{U}{t} = \frac{2.25 \times 10^{-3}}{10^{-4}} = \mathbf{22.5\ W}.$$

Example 73. *A 800 pF capacitor is charged by a 100 V battery. After some time the battery is disconnected. The capacitor is then connected to another 800 pF capacitor. What is the electrostatic energy stored ?* [CBSE F 09]

Solution. Here $C_1 = C_2 = 800\ \text{pF} = 8 \times 10^{-10}$ F,

$$V_1 = 100\ \text{V}, \quad V_2 = 0$$

Common potential,

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{8 \times 10^{-10} \times 100 + 0}{8 \times 10^{-10} + 8 \times 10^{-10}} = 50\ \text{V}$$

$$U_f = \frac{1}{2}(C_1 + C_2) V^2$$

$$= \frac{1}{2}(8 \times 10^{-10} + 8 \times 10^{-10}) \times (50)^2 = \mathbf{2 \times 10^{-6}\ J}$$

Example 74

(*i*) *A 900 pF capacitor is charged by a 100 V battery. How much electrostatic energy is stored by the capacitor ?*

(*ii*) *The capacitor is disconnected from the battery and connected to another 900 pF capacitor. What is the electrostatic energy stored by the system ?*

(*iii*) *Where has the remainder of the energy gone ?* [NCERT ; CBSE OD 90]

Solution. (*i*) The charge on the capacitor is

$$q = CV = 900 \times 10^{-12}\ \text{F} \times 100\ \text{V} = 9 \times 10^{-8}\ \text{C}$$

The energy stored by the capacitor is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}qV = \frac{1}{2} \times 9 \times 10^{-8}\text{C} \times 100\ \text{V}$$

$$= \mathbf{4.5 \times 10^{-6}\ J.}$$

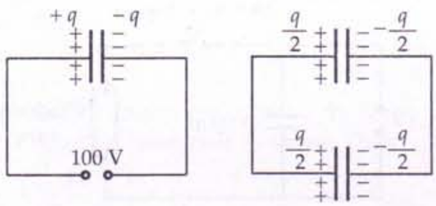


Fig. 2.111

(*ii*) In the steady situation, the two capacitors have their positive plates at the same potential, and their negative plates at the same potential. Let the common potential difference be V'. The charge on each capacitor is then $q' = CV'$. By charge conservation, $q' = q/2$.

$\therefore$ Total energy of the system

$$= 2 \times \frac{1}{2}q'V' = q'V' = q' \cdot \frac{q'}{C}$$

$$= \frac{1}{4} \cdot \frac{q^2}{C} = \frac{1}{4} \cdot qV = \frac{1}{2} \times \frac{1}{2}qV \quad \left[\because\ q' = \frac{q}{2} \text{ and } \frac{q}{C} = V\right]$$

$$= \frac{1}{2} \times 4.5 \times 10^{-6}\ \text{J} = \mathbf{2.25 \times 10^{-6}\ J.}$$

(*iii*) There is a transient period before the system settles to the situation (*ii*). During this period, a transient current flows from the first capacitor to the second. Energy is lost during this time in the form of heat and electromagnetic radiation.

Example 75. *A capacitor is charged to potential V_1. The power supply is disconnected and the capacitor is connected in parallel to another uncharged capacitor.*

(*i*) *Derive the expression for the common potential of the combination of capacitors.*

(*ii*) *Show that total energy of the combination is less than the sum of the energy stored in them before they are connected.* [CBSE OD 15]

Solution. (*i*) Let C_1 and C_2 be the capacitances of the two capacitors and V be their common potential. Then

$$V = \frac{\text{Total charge}}{\text{Total capacitance}} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

or $$V = \frac{C_1V_1}{C_1 + C_2} \qquad [\because V_2 = 0]$$

(*ii*) Energy stored in the capacitors before connection,

$$U_i = \frac{1}{2}C_1V_1^2$$

Total energy after connection,

$$U_f = \frac{1}{2}(C_1 + C_2)V^2$$

$$= \frac{1}{2}(C_1 + C_2)\frac{C_1^2V_1^2}{(C_1 + C_2)^2}$$

$$= \frac{1}{2}\frac{C_1^2V_1^2}{C_1 + C_2} = \left(\frac{C_1}{C_1 + C_2}\right)U_i$$

Clearly, $U_f < U_i$

Hence total energy of the combination is less than the sum of the energy stored in the capacitors before they are connected.

Example 76. *Two capacitors of unknown capacitances C_1 and C_2 are connected first in series and then in parallel across a battery of 100 V. If the energy stored in the two combinations is 0.045 J and 0.25 J respectively, determine the values of C_1 and C_2. Also calculate the charge on each capacitor in parallel combination.* [CBSE D 15]

Solution. For series combination, we have

$$U = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}V^2$$

$$\therefore \quad 0.045 = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2} \times (100)^2 \qquad ...(i)$$

For parallel combination, we have

$$U = \frac{1}{2}(C_1 + C_2)V^2$$

$$\therefore \quad 0.25 = \frac{1}{2}(C_1 + C_2) \times (100)^2$$

or $$C_1 + C_2 = 0.5 \times 10^{-4} \qquad ...(ii)$$

From (*i*), $$0.045 = \frac{1}{2} \times \frac{C_1C_2}{0.5 \times 10^{-4}} \times (100)^2$$

or $$C_1C_2 = 0.045 \times 10^{-8}$$

Now $$(C_1 - C_2)^2 = (C_1 + C_2)^2 - 4C_1C_2$$

$$= (0.5 \times 10^{-4})^2 - 4 \times 0.045 \times 10^{-8}$$

$$= (0.25 - 0.180) \times 10^{-8} = 0.07 \times 10^{-8}$$

$$\therefore \quad C_1 - C_2 = \sqrt{0.07} \times 10^{-4} = 0.26 \times 10^{-4} \qquad ...(iii)$$

On solving (*ii*) and (*iii*), we get

$$C_1 = \mathbf{0.38 \times 10^{-4}\ F} \text{ and } C_2 = \mathbf{0.12 \times 10^{-4}\ F}$$

Charges on capacitors C_1 and C_2 in parallel combination are :

$$Q_1 = C_1 V = 0.38 \times 10^{-4} \times 100\,\text{C} = \mathbf{0.38 \times 10^{-2}\ C}$$

$$Q_2 = C_2 V = 0.12 \times 10^{-4} \times 100\,\text{C} = \mathbf{0.12 \times 10^{-2}\ C.}$$

Example 77. *A capacitor of capacitance 6 μF is charged to a potential of 150 V. Its potential falls to 90 V, when another capacitor is connected to it. Find the capacitance of the second capacitor and the amount of energy lost due to the connection.*

Solution. Here $C_1 = 6\,\mu\text{F}, \quad V_1 = 150\text{ V}, \quad V_2 = 0,$

$$V = 90\text{ V}, \quad C_2 = ?$$

Common potential,

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

or
$$90\text{ V} = \frac{6 \times 10^{-6} \times 150 + 0}{6 \times 10^{-6} + C_2}$$

or
$$C_2 + 6 \times 10^{-6} = \frac{6 \times 10^{-6} \times 150}{90} = 10 \times 10^{-6}$$

or
$$C_2 = 4 \times 10^{-6}\text{ F} = \mathbf{4\ \mu F.}$$

Initial energy stored,

$$U_i = U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times 6 \times 10^{-6} \times (150)^2$$

$$= 6.75 \times 10^{-2}\text{ J}$$

Final energy stored,

$$U_f = \frac{1}{2}(C_1 + C_2) V^2$$

$$= \frac{1}{2}(6+4) \times 10^{-6} \times (90)^2 = 4.05 \times 10^{-2}\text{ J}$$

The loss of energy on connecting the two capacitors,

$$\Delta U = U_i - U_f = (6.75 - 4.05) \times 10^{-2}$$

$$= 2.7 \times 10^{-2}\text{ J} = \mathbf{0.027\ J.}$$

Example 78. *A battery of 10 V is connected to a capacitor of capacity 0.1 F. The battery is now removed and this capacitor is connected to a second uncharged capacitor. If the charge distributes equally on these two capacitors, find the total energy stored in the two capacitors. Further, compare this energy with the initial energy stored in the first capacitor.* [Roorkee 96]

Solution. Initial energy stored in the first capacitor is

$$U_i = \frac{1}{2} CV^2 = \frac{1}{2} \times 0.1 \times (10)^2 = 5.0\text{ J}$$

When the first capacitor is connected to the second uncharged capacitor, the charge distributes equally. This implies that the capacitance of second capacitor is also C. The voltage across each capacitor is now $V/2$. The final total energy stored in the two capacitors is

$$U_f = \frac{1}{2} C\left(\frac{V}{2}\right)^2 + \frac{1}{2} C\left(\frac{V}{2}\right)^2 = \frac{1}{4} CV^2$$

$$= 2.5\text{ J}$$

$$\therefore \quad \frac{U_f}{U_i} = \frac{2.5}{5.0} = \frac{1}{2} = \mathbf{1:2.}$$

Problems for Practice

1. A capacitor charged from a 50 V d.c. supply is found to have charge of 10 μC. What is the capacitance of the capacitor and how much energy is stored in it ? [ISCE 93]
(Ans. 0.2 μF, 2.5×10^{-4} J)
2. For flash pictures, a photographer uses a capacitor of 30 μF and a charger that supplies 3×10^3 V. Find the charge and energy expended in joule for each flash. (Ans. 9×10^{-2} C, 135 J)
3. An electronic flash lamp has 10 capacitors, each 10 μF, connected in parallel. The lamp is operated at 100 volt. How much energy will be radiated in the flash ? (Ans. 0.5 J)
4. Three capacitors of capacitances 10 μF, 20 μF and 30 μF are connected in parallel to a 100 V battery as shown in Fig. 2.112. Calculate the energy stored in the capacitors. [ISCE 94]
(Ans. 0.3 J)

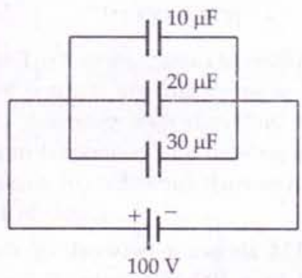


Fig. 2.112

5. A variable capacitor is kept connected to a 10 V battery. If the capacitance of the capacitor is changed from 7 μF to 3 μF, what is the change in the energy ? What happens to this energy ? [ISCE 96]
(Ans. 2×10^{-4} J, decrease in energy)
6. The plates of a parallel plate capacitor have an area of 100 cm^2 each and are separated by 2.5 mm. The capacitor is charged to 200 V. Calculate the energy stored in the capacitor. [Punjab 96]
(Ans. 7.08×10^{-7} J)
7. A 80 μF capacitor is charged by a 50 V battery. The capacitor is disconnected from the battery and then

connected across another unchanged 320 μF capacitor. Calculate the charge on the second capacitor. **[CBSE D 94 C]**

(Ans. 3.2×10^{-3}C)

8. Find the total energy stored in the capacitors in the network shown below. **[CBSE D 04]**

(Ans. 3.6×10^{-5} J)

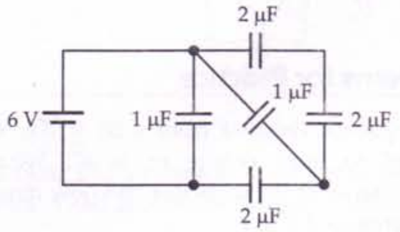


Fig. 2.113

9. A 10 μF capacitor is charged by a 30 V d.c. supply and then connected across an uncharged 50 μF capacitor. Calculate (*i*) the final potential difference across the combination, and (*ii*) the initial and final energies. How will you account for the difference in energy ? **[CBSE OD 04]**

[Ans. (*i*) 5V, (*ii*) $U_i = 4.5\times10^{-3}$ J, $U_f = 0.75\times10^{-3}$ J]

10. Net capacitance of three identical capacitors in series is 1 μF. What will be their net capacitance if connected in parallel ?
Find the ratio of energy stored in the two configurations if they are both connected to the same source. **[CBSE OD 11]** (Ans. 9 μF, 1 : 9)

11. Two capacitors of capacitances 25 μF and 100 μF are connected in series and are charged by a battery of 120 V. The battery is then removed. The capacitors are now separated and connected in parallel. Find (*i*) p.d. across each capacitor (*ii*) energy-loss in the process. (Ans. 38.4 V, 0.05184 J)

12. Figure 2.114 shows a network of five capacitors connected to a 100 V supply. Calculate the total charge and energy stored in the network. **[CBSE Sample Paper 08]**

(Ans. 4×10^{-4}C, 0.02 J)

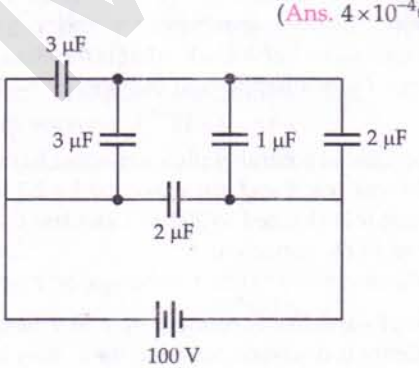


Fig. 2.114

13. Two capacitors are in parallel and the energy stored is 45 J, when the combination is raised to potential of 3000 *V*. With the same two capacitors in series, the energy stored is 4.05 J for the same potential. What are their individual capacitances ?

(Ans. 9 μF, 1 μF)

14. Find the ratio of the potential differences that must be applied across the parallel and the series combination of two capacitors C_1 and C_2 with their capacitances in the ratio 1 : 3 so that the energy stored in the two cases, becomes the same. **[CBSE F 10]**

(Ans. $\sqrt{3}:4$)

HINTS

1. $C = \frac{q}{V} = \frac{10\,\mu C}{50\,V} = 0.2\,\mu F.$

Energy stored,

$U = \frac{1}{2}CV^2 = \frac{1}{2}\times0.2\times10^{-6}\times(50)^2 = \mathbf{2.5\times10^{-4}\ J.}$

2. Here $C = 30\,\mu F = 3\times10^{-5}F,\ V = 3\times10^3$ V

Charge, $q = CV = 3\times10^{-5}\times3\times10^3\ C = \mathbf{9\times10^{-2}\ C}$

Energy, $U = \frac{1}{2}CV^2 = \frac{1}{2}\times3\times10^{-5}\times9\times10^6$

$= \mathbf{135\ J.}$

3. Total equivalent capacitance,

$C = 10\times10\,\mu F = 100\,\mu F = 10^{-4}F$

Energy radiated

$= \frac{1}{2}CV^2 = \frac{1}{2}\times10^{-4}\times(100)^2 = \mathbf{0.5\ J.}$

4. $C = C_1 + C_2 + C_3 = 10 + 20 + 30 = 60\,\mu F$

$= 60\times10^{-6}$ F

$U = \frac{1}{2}CV^2 = \frac{1}{2}\times60\times10^{-6}\times(100)^2 = \mathbf{0.3\ J.}$

5. Here $C_i = 7\mu F = 7\times10^{-6}F,\ V = 10$ V

$U_i = \frac{1}{2}C_iV^2 = \frac{1}{2}\times7\times10^{-6}\times(10)^2 = 3.5\times10^{-4}$ J

Again, $C_f = 3\mu F = 3\times10^{-6}$ F, $V = 10$ V

$U_f = \frac{1}{2}C_fV^2 = \frac{1}{2}\times3\times10^{-6}\times(10)^2 = 1.5\times10^{-4}$J

Decrease in energy $= U_i - U_f = \mathbf{2.0\times10^{-4}\ J.}$

Energy is lost as heat and electromagnetic radiation.

6. Here $A = 100\text{cm}^2 = 10^{-2}\text{m}^2$,

$d = 2.5\text{ mm} = 2.5\times10^{-3}\text{m},\ V = 200$ V

$U = \frac{1}{2}CV^2 = \frac{1}{2}.\frac{\varepsilon_0 A}{d}.V^2$

$= \frac{1}{2}\times\frac{8.85\times10^{-12}\times10^{-2}}{2.5\times10^{-3}}(200)^2$

$= \mathbf{7.08\times10^{-7}\ J.}$

8. The two $2\,\mu F$ capacitors on the right side are in series, their equivalent capacitance $= \frac{2 \times 2}{2+2} = 1\,\mu F$

This $1\,\mu F$ capacitance is in parallel with the central $1\,\mu F$ capacitor. Their equivalent capacitance

$$= 1 + 1 = 2\,\mu F$$

This $2\,\mu F$ capacitance is in series with the $2\,\mu F$ capacitor at the bottom. Their equivalent capacitance

$$= \frac{2 \times 2}{2+2} = 1\,\mu F$$

Finally, $1\,\mu F$ capacitance is in parallel with the left out $1\,\mu F$ capacitor. The equivalent capacitance is

$$C = 1 + 1 = 2\,\mu F = 2 \times 10^{-6}\ F$$

$$V = 6\ V$$

$$\therefore \quad U = \frac{1}{2} CV^2 = \frac{1}{2} \times 2 \times 10^{-6} \times (6)^2$$

$$= \mathbf{3.6 \times 10^{-5}\ J.}$$

9. Here $C_1 = 10\,\mu F = 10 \times 10^{-6} F$, $V_1 = 30\ V$,

$C_2 = 50\,\mu F = 50 \times 10^{-6} F$, $V_2 = 0$

(*i*) Common potential,

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{10 \times 10^{-6} \times 30 + 0}{(10+50) \times 10^{-6}} = \mathbf{5\ V.}$$

(*ii*) Initial electrostatic energy of $10\,\mu F$ capacitor,

$$U_i = \frac{1}{2} C_1 V_1^2$$

$$= \frac{1}{2} \times 10 \times 10^{-6} \times (30)^2 = \mathbf{4.5 \times 10^{-3}\ J}$$

Final electrostatic energy of the combination,

$$U_f = \frac{1}{2}(10+50) \times 10^{-6} \times (5)^2 = \mathbf{0.75 \times 10^{-3}\ J}$$

Loss in energy $= U_i - U_f = 3.75 \times 10^{-3}$ J

The difference in energy is lost in the form of heat and electromagnetic radiation as the charge flows from first capacitor to second capacitor.

10. Here $C_s = \frac{C}{3} = 1\,\mu F$

$$\therefore \quad C = 3\,\mu F$$

$$C_p = 3C = \mathbf{9\ \mu F}$$

$$\frac{U_s}{U_p} = \frac{\frac{1}{2}C_sV^2}{\frac{1}{2}C_pV^2} = \frac{C_s}{C_p} = \frac{1}{9} = \mathbf{1 : 9}$$

11. (*i*) Equivalent capacitance in series,

$$C = \frac{25 \times 100}{25 + 100} = 20\,\mu F$$

Charge on each capacitor in series,

$$q = CV = 20\,\mu F \times 120\ V = 2400\,\mu C$$

Equivalent capacitance in parallel,

$$C' = 25 + 100 = 125\,\mu F$$

Total charge,

$$q' = 2400 + 2400 = 4800\,\mu C$$

$\therefore$ P.D. across each capacitor,

$$V' = \frac{q'}{C'} = \frac{4800\,\mu C}{125\,\mu F} = \mathbf{38.4\ V.}$$

(*ii*) In series,

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times 20 \times 10^{-6} \times (120)^2$$

$$= 0.144\ J$$

In parallel,

$$U' = \frac{1}{2} C' V'^2 = \frac{1}{2} \times 125 \times 10^{-6} \times (38.4)^2 = 0.09216\ J$$

$\therefore$ Energy loss

$$= U - U' = 0.144 - 0.09216 = \mathbf{0.05184\ J.}$$

12. The equivalent circuit diagram for the given network is shown below :

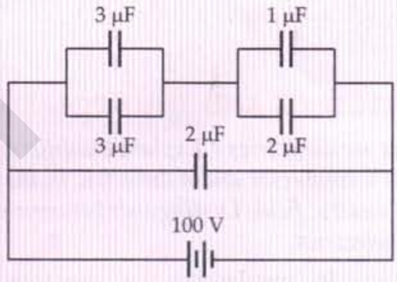


Fig. 2.115

Two $3\,\mu F$ capacitors in parallel. The equivalent capacitance,

$$C_1 = 3 + 3 = 6\,\mu F$$

The $1\,\mu F$ capacitor and a $2\,\mu F$ capacitor are in parallel. Their equivalent capacitance,

$$C_2 = 1 + 2 = 3\,\mu F$$

Then C_1 and C_2 form a series combination of equivalent capacitance,

$$C_{12} = \frac{6 \times 3}{6+3} = 2\,\mu F$$

This combination is in parallel with the fifth capacitor of $2\,\mu F$.

$\therefore$ Net capacitance, $C = 2 + 2 = 4\,\mu F$

Total charge,

$$q = CV = 4 \times 10^{-6} \times 100 = \mathbf{4 \times 10^{-4}\ C}$$

Total energy stored,

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times 4 \times 10^{-6} \times (100)^2 = \mathbf{0.02\ J.}$$

14. Given $\frac{C_1}{C_2} = \frac{1}{3}$

Now $U_p = U_s$

or $\frac{1}{2} C_p V_p^2 = \frac{1}{2} C_s V_s^2$

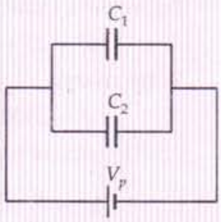
or $\frac{V_p^2}{V_s^2} = \frac{C_s}{C_p}$

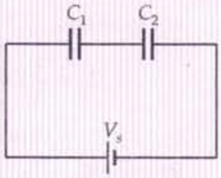
$$= \frac{\frac{C_1 C_2}{C_1 + C_2}}{C_1 + C_2}$$

$$= \frac{C_1 C_2}{(C_1 + C_2)^2}$$

$$= \frac{\frac{C_1}{C_2}}{\left(\frac{C_1}{C_2} + 1\right)^2} = \frac{\frac{1}{3}}{\left(\frac{1}{3} + 1\right)^2} = \frac{3}{16}$$

$\therefore \quad \frac{V_p}{V_s} = \frac{\sqrt{3}}{4} = \sqrt{3} : 4$





2.27 DIELECTRICS AND THEIR POLARIZATION

43. *What are dielectrics ? Explain the difference in the behaviour of a conductor and a dielectric in the presence an external electric field. Distinguish between polar and non-polar dielectrics.*

Dielectrics. In insulators, the electrons remain attached to the individual atoms or molecules. However, these electrons can suffer small movements within the atoms or molecules under the influence of an external electric field. The net effect of these microscopic movements gives rise to some important electric properties to such materials. In view of these electrical properties, insulators are called *dielectrics*.

A ***dielectric*** *is a substance which does not allow the flow of charges through it but permits them to exert electrostatic forces on one another through it. A* ***dielectric*** *is essentially an insulator which can be polarised through small localised displacements of its charges.*

Examples. Glass, wax, water, air, wood, rubber, stone, plastic, etc.

Difference in the behaviour of a conductor and a dielectric in the presence of an external electric field. Dielectrics have negligibly small number of charge carriers as compared to conductors.

In a conductor, the external field E_0 moves the free charge carriers inducing field E_{ind} in the opposite direction of E_0. The process continues until the two fields cancel each other and the net electric field in the conductor becomes zero.

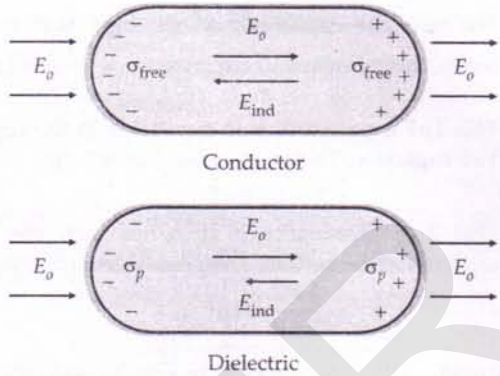


Fig. 2.116 Difference in the behaviour of a conductor and a dielectric in an external electric field.

In a dielectric, the external field E_0 induces dipole moment by stretching or re-orienting the molecules of the dielectric. The induced dipole moment sets up an electric field E_{ind} which opposes E_0 but does not exactly cancel this field. It only reduces it.

Polar and non-polar dielectrics. A dielectric may consist of either polar or non-polar molecules. *A molecule in which the centre of mass of positive charges (protons) does not coincide with the centre of mass of negative charges (electrons) is called a polar molecule.*

The dielectrics made of polar molecules are called *polar dielectrics*. The polar molecules have unsymmetrical shapes. They have permanent dipole moments of the order of 10^{-30} Cm. For example, a water molecule has a bent shape with its two O - H bonds inclined at an angle of 105° as shown in Fig. 2.117. It has a very large dipole moment of 6.1×10^{-30} Cm. Some other polar molecules are HCl, NH_3, CO, CH_3OH, etc.

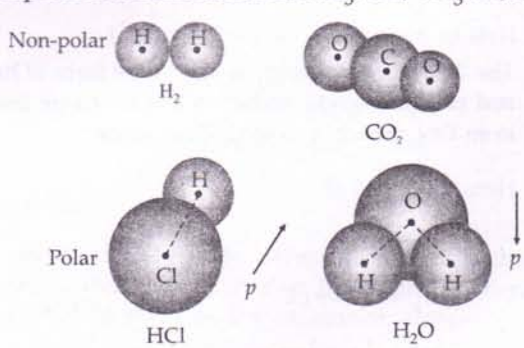


Fig. 2.117 Some polar and non-polar molecules.

A molecule in which the centre of mass of positive charges coincides with the centre of mass of negative charges is called a non-polar molecule. The dielectrics made of non-polar molecules are called *non-polar dielectrics*. Non-polar molecules have symmetrical shapes. They have normally zero dipole moment. Examples of non-polar molecules are H_2, N_2, O_2, CO_2, CH_4, etc.

44. *How does a dielectric develop a net dipole moment in an external electric field when it has (i) non-polar molecules and (ii) polar molecules ?*

Polarization of a non-polar dielectric in an external electric field. In the absence of any electric field, the centres of positive and negative charges of the molecules of a non-polar dielectric coincide, as shown in Fig. 2.118(*a*)(*i*). The dipole moment of each molecule is zero. In the presence of an external electric field E_0, the centres of positive charges are displaced in the direction of external field while the centres of negative charges are displaced in the opposite direction. The displacement of the charges stops when the force exerted on them by the external field is balanced by the restoring force due to the internal fields in the molecules. This induces dipole moment in each molecule *i.e.,* each non-polar molecule becomes an induced dipole. The induced dipole moments of different molecules add up giving a net dipole moment to the dielectric in the direction of the external field, as shown in Fig. 2.118(*a*)(*ii*).

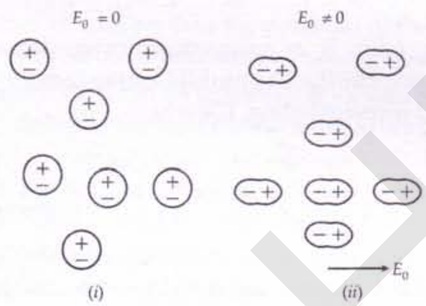


Fig. 2.118 (*a*) Polarization of a non-polar dielectric in an external electric field.

Polarization of a polar dielectric in an external electric field. The molecules of a polar dielectric have permanent dipole moments. In the absence of any external electric field, the dipole moments of different molecules are randomly oriented due to thermal agitation in the material, as shown in Fig. 2.118(*b*)(*i*). So the total dipole moment is zero. When an external field is applied, the dipole moments of different molecules

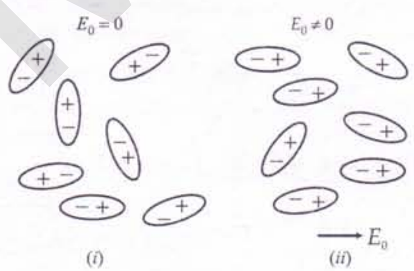


Fig. 2.118 (*b*) Polarization of a polar dielectric in an external electric field.

tend to align with the field. As a result, there is a net dipole moment in the direction of the field, as shown in Fig. 2.118(*b*)(*ii*). The extent of polarisation depends on relative values of two opposing energies :

1. The potential energy of the dipole in the external field which tends to align the dipole with the field.
2. Thermal energy of agitation which tends to randomise the alignment of the dipole.

Hence both *polar and non-polar dielectrics develop a net dipole moment in the presence of an external electric field. This fact is called* ***polarization of the dielectric****.*

The ***polarization*** $\vec{P}$ *is defined as the dipole moment per unit volume* and its magnitude is usually referred to as the ***polarization density***. The direction of $\vec{P}$ is same as that of the external field $\vec{E}_0$.

45. *Explain why the polarization of dielectric reduces the electric field inside the dielectric. Hence define dielectric constant.*

Reduction of electric field by the polarization of a dielectric. Consider a rectangular dielectric slab placed in a uniform electric field $\vec{E}_0$ acting parallel to two of its faces, as shown in Fig. 2.119(*a*). Its molecular dipoles

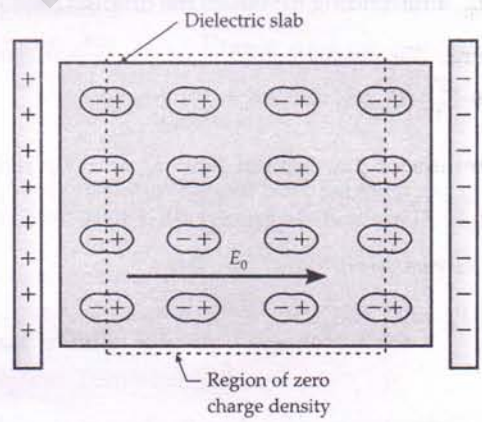


Fig. 2.119 (*a*) Polarization of a dielectric.

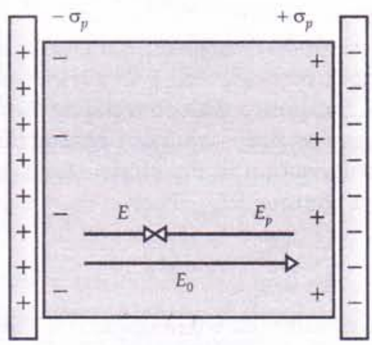


Fig. 2.119 (*b*) Reduced field in a dielectric, $E = E_0 - E_p$.

align themselves in the direction of $\vec{E}_0$. This results in uniform polarization of the dielectric, *i.e.*, every small volume of the slab has a dipole moment in the direction of $\vec{E}_0$. The positive charges of the dipoles of first vertical column cancel the negative charges of the dipoles of second column and so on. Thus the volume charge density in the interior of the slab is zero. However, there is a net uncancelled negative charge on the left face and uncancelled positive charge on the right face of the slab.

The uncancelled charges are the induced surface charges due to the external field $\vec{E}_0$. Since the slab as a whole remains electrically neutral, the magnitude of the positive induced surface charge is equal to that of the negative induced surface charge.

Thus *the polarized dielectric is equivalent to two charged surfaces with induced surface charge densities* $\pm\,\sigma_p$.

Reduced field inside a dielectric and dielectric constant. In case of a homogeneous and isotropic dielectric, the induced surface charges set up an electric field $\vec{E}_p$ (field due to polarization) inside the dielectric in a direction opposite to that of external field $\vec{E}_0$, thus tending to reduce the original field in the dielectric. *The resultant field* $\vec{E}$ *in the dielectric will be equal to* $\vec{E}_0 - \vec{E}_p$ *and directed in the direction of* $\vec{E}_0$.

The ratio of the original field $\vec{E}_0$ and the reduced field $\vec{E}_0 - \vec{E}_p$ in the dielectric is called *dielectric constant* (κ) or *relative permittivity* (ε_r). Thus

$$\kappa = \frac{\vec{E}_0}{\vec{E}} = \frac{\vec{E}_0}{\vec{E}_0 - \vec{E}_p}$$

46. *Define polarisation density. How is it related to the induced surface charge density ?*

Polarisation density. *The induced dipole moment developed per unit volume of a dielectric when placed in an external electric field is called polarisation density. It is denoted by P.* Suppose a dielectric slab of surface area A and thickness d acquires a surface charge density $\pm\,\sigma_p$ due to its polarisation in the electric field and its two faces acquire charges $\pm\,Q_p$. Then

$$\sigma_p = \frac{Q_p}{A}$$

We can consider the whole dielectric slab as a large dipole having dipole moment equal to $Q_p\,d$. The dipole moment per unit volume or the polarisation density will be

$$P = \frac{\text{dipole moment of dielectric}}{\text{volume of dielectric}}$$

$$= \frac{Q_p\,d}{Ad} = \frac{Q_p}{A} = \sigma_p$$

Thus the polarisation density may be defined as the charge induced per unit surface area.

Obviously, a uniformly polarised dielectric with uniform polarisation density P can be replaced by two surface layers (perpendicular to $\vec{P}$) of surface charge densities $\pm\,\sigma_p$, and zero charge density in the interior.

47. *Define electric susceptibility. Deduce the relation between dielectric constant and electric susceptibility.*

Electric susceptibility. If the field $\vec{E}$ is not large, then the polarisation $\vec{P}$ is proportional to the resultant field $\vec{E}$ existing in the dielectric, *i.e.*,

$$\vec{P} \propto \vec{E} \qquad \text{or} \qquad \vec{P} = \varepsilon_0\,\chi\,\vec{E}$$

where χ (chi) is a proportionality constant called *electric susceptibility*. The multiplicative factor ε_0 is used to keep χ dimensionless. Clearly,

$$\chi = \frac{\vec{P}}{\varepsilon_0\,\vec{E}}$$

Thus the ratio of the polarisation to ε_0 *times the electric field is called the electric susceptibility of the dielectric.* Like P, it also describes the electrical behaviour of a dielectric. The dielectrics with constant χ are called *linear dielectrics*.

Relation between κ and χ. The net electric field in a polarised dielectric is

$$\vec{E} = \vec{E}_0 - \vec{E}_p$$

But $$\vec{E}_p = \frac{\sigma_p}{\varepsilon_0} = \frac{P}{\varepsilon_0}$$

$\therefore$ $$\vec{E} = \vec{E}_0 - \frac{P}{\varepsilon_0}$$

or $$\vec{E} = \vec{E}_0 - \frac{\varepsilon_0\,\chi\,\vec{E}}{\varepsilon_0} \qquad [\vec{P} = \varepsilon_0\,\chi\,\vec{E}]$$

Dividing both sides by $\vec{E}$, we get

$$1 = \frac{\vec{E}_0}{\vec{E}} - \chi$$

or $$1 = \kappa - \chi \qquad \text{or} \qquad \kappa = 1 + \chi$$

2.28 DIELECTRIC STRENGTH

48. *What do you mean by dielectric strength of a dielectric ?*

Dielectric strength. When a dielectric is placed in a very high electric field, the outer electrons may get detached from their parent atoms. The dielectric then behaves like a conductor. This phenomenon is called *dielectric breakdown.*

The maximum electric field that can exist in a dielectric without causing the breakdown of its insulating property is called ***dielectric strength*** *of the material.*

The unit of dielectric strength is same as that of electric field *i.e.*, Vm^{-1}. But the more common practical unit is kV mm^{-1}.

Table 2.1 Dielectric constants and dielectric strengths of some common dielectrics.

Dielectric	Dielectric constant	Dielectric strength in kV mm^{-1}
Vacuum	1.00000	∞
Air	1.00054	0.8
Water	81	—
Paper	3.5	14
Pyrex glass	4.5	13
Mica	5.4	160
Porcelain	6.5	4

2.29 CAPACITANCE OF A PARALLEL PLATE CAPACITOR WITH A DIELECTRIC SLAB

49. *Deduce the expression for the capacitance of a parallel plate capacitor when a dielectric slab is inserted between its plates. Assume the slab thickness less than the plate separation.*

Capacitance of a parallel plate capacitor with a dielectric slab. The capacitance of a parallel plate capacitor of plate area A and plate separation d with vacuum between its plates is given by

$$C_0 = \frac{\varepsilon_0 A}{d}$$

Suppose initially the charges on the capacitor plates are $\pm Q$. Then the uniform electric field set up between the capacitor plates is

$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

When a dielectric slab of thickness $t < d$ is placed between the plates, the field E_0 polarises the dielectric. This induces charge $-Q_p$ on the upper surface and

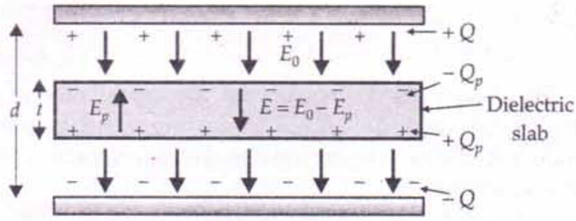


Fig. 2.120 A dielectric slab placed in a parallel plate capacitor.

$+Q_p$ on the lower surface of the dielectric. These induced charges set up a field E_p inside the dielectric in the opposite direction of $\vec{E}_0$. The induced field is given by

$$E_p = \frac{\sigma_p}{\varepsilon_0} = \frac{P}{\varepsilon_0} \quad \left[\sigma_p = \frac{Q}{A} = P, \text{ polarisation density}\right]$$

The net field inside the dielectric is

$$E = E_0 - E_p = \frac{E_0}{\kappa} \qquad \left[\because \frac{E_0}{E_0 - E_p} = \kappa\right]$$

where κ is the dielectric constant of the slab. So between the capacitor plates, the field E exists over a distance t and field E_0 exists over the remaining distance $(d-t)$. Hence the potential difference between the capacitor plates is

$$V = E_0(d-t) + Et = E_0(d-t) + \frac{E_0}{\kappa}t \qquad \left[\because \frac{E_0}{E} = \kappa\right]$$

$$= E_0\left(d - t + \frac{t}{\kappa}\right) = \frac{Q}{\varepsilon_0 A}\left(d - t + \frac{t}{\kappa}\right)$$

The capacitance of the capacitor on introduction of dielectric slab becomes

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d - t + \frac{t}{\kappa}}.$$

Special Case If the dielectric fills the entire space between the plates, then $t = d$, and we get

$$C = \frac{\varepsilon_0 A}{d} \cdot \kappa = \kappa\, C_0$$

Thus *the capacitance of a parallel plate capacitor increases* κ *times when its entire space is filled with a dielectric material.*

Clearly, $\kappa = \dfrac{C}{C_0}$

Dielectric constant

$$= \frac{\text{Capacitance with dielectric between two plates}}{\text{Capacitance with vacuum between two plates}}$$

Thus the ***dielectric constant*** *of a dielectric material may be defined as the ratio of the capacitance of a capacitor completely filled with that material to the capacitance of the same capacitor with vacuum between its plates.*

2.30 CAPACITANCE OF A PARALLEL PLATE CAPACITOR WITH A CONDUCTING SLAB

50. *Deduce the expression for the capacitance of a parallel plate capacitor when a conducting slab is inserted between its plates. Assume the slab thickness less than the plate separation.*

Capacitance of a parallel plate capacitor with a conducting slab. Consider a parallel plate capacitor of plate area A and plate separation d. If the space between the plates is vacuum, its capacitance is given by

$$C_0 = \frac{\varepsilon_0 A}{d}$$

Suppose initially the charges on the capacitor plates are $\pm Q$. Then the uniform electric field set up between the capacitor plates is

$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

where σ is the surface charge density. The potential difference between the capacitor plates will be

$$V_0 = E_0 d = \frac{Qd}{A\varepsilon_0}$$

When a conducting slab of thickness $t < d$ is placed between the capacitor plates, free electrons flow inside it so as to reduce the field to zero inside the slab, as shown in Fig. 2.121. Charges $-Q$ and $+Q$ appear on the upper and lower faces of the slab. Now the electric field exists only in the vacuum regions between the plates of the capacitor on the either side of the slab, *i.e.*, the field exists only in thickness $d - t$, therefore, potential difference between the plates of the capacitor is

$$V = E_0(d-t) = \frac{Q}{A\varepsilon_0}(d-t)$$

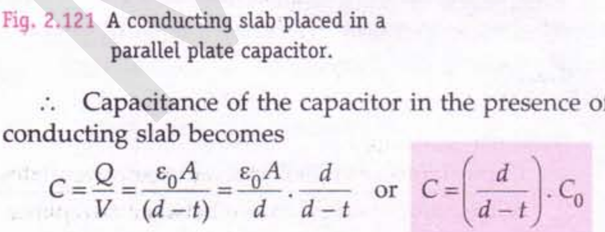


Fig. 2.121 A conducting slab placed in a parallel plate capacitor.

∴ Capacitance of the capacitor in the presence of conducting slab becomes

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{(d-t)} = \frac{\varepsilon_0 A}{d}\cdot\frac{d}{d-t} \quad \text{or} \quad C = \left(\frac{d}{d-t}\right)\cdot C_0$$

Clearly, $C > C_0$. Thus the introduction of a conducting slab of thickness t in a parallel plate capacitor increases its capacitance by a factor of $\frac{d}{d-t}$.

2.31 USES OF CAPACITORS

51. *Mention some important uses of capacitors.*

Uses of capacitors. Capacitors are very useful circuit elements in any of the electric and electronic circuits. Some of their uses are

1. To produce electric fields of desired patterns, *e.g.*, for Millikan's experiment.
2. In radio circuits for tuning.
3. In power supplies for smoothing the rectified current.
4. For producing rotating magnetic fields in induction motors.
5. In the tank circuit of oscillators.
6. They store not only charge, but also energy in the electric field between their plates.

2.32 EFFECT OF DIELECTRIC ON VARIOUS PARAMETERS

52. *A parallel-plate capacitor is charged by a battery which is then disconnected. A dielectric slab is then inserted to fill the space between the plates. Explain the changes, if any, that occur in the values of (i) charge on the plates, (ii) electric field between the plates, (iii) p.d. between the plates, (iv) capacitance and (v) energy stored in the capacitor.*

Effect of dielectric when the battery is kept disconnected from the capacitor. Let Q_0, C_0, V_0, E_0 and U_0 be the charge, capacitance, potential difference, electric field and energy stored respectively before the dielectric slab is inserted. Then

$$Q_0 = C_0 V_0, \; E_0 = \frac{V_0}{d}, \; U_0 = \frac{1}{2} C_0 V_0^2$$

(*i*) **Charge.** The charge on the capacitor plates *remains* Q_0 because the battery has been disconnected before the insertion of the dielectric slab.

(*ii*) **Electric field.** When the dielectric slab is inserted between the plates, the induced surface charge on the dielectric *reduces the field* to a new value given by

$$E = \frac{E_0}{\kappa}$$

(*iii*) **Potential difference.** The reduction in the electric field results in the *decrease in potential difference.*

$$V = Ed = \frac{E_0 d}{\kappa} = \frac{V_0}{\kappa}$$

(*iv*) **Capacitance.** As a result of the decrease in potential difference, the *capacitance increases* κ times.

$$C = \frac{Q_0}{V} = \frac{Q_0}{V_0/\kappa} = \kappa\frac{Q_0}{V_0} = \kappa C_0$$

(*v*) **Energy stored.** The energy stored decreases by a factor of κ.

$$U = \frac{1}{2} CV^2 = \frac{1}{2} (\kappa C_0) \left(\frac{V_0}{\kappa} \right)^2 = \frac{1}{\kappa} \cdot \frac{1}{2} C_0 V_0^2 = \frac{U_0}{\kappa}.$$

53. *A parallel plate capacitor is charged by a battery. When battery remains connected, a dielectric slab is inserted between the plates. Explain what changes, if any, occur in the values of (i) p.d. between the plates, (ii) electric field between the plates, (iii) capacitance, (iv) charge on the plates and (v) energy stored in the capacitor ?*

Effect of dielectric when battery remains connected across the capacitor. Let Q_0, C_0, V_0, E_0 and U_0 be the charge, capacitance, potential difference, electric field and energy stored respectively, before the introduction of the dielectric slab. Then

$$Q_0 = C_0 V_0, \; E_0 = \frac{V_0}{d}, \; U_0 = \frac{1}{2} C_0 V_0^2$$

(*i*) **Potential difference.** As the battery remains connected across the capacitor, so the *potential difference remains constant* at V_0 even after the introduction of dielectric slab.

(*ii*) **Electric field.** As the potential difference remains unchanged, so the electric field E_0 between the capacitor plates remains unchanged.

$$E = \frac{V}{d} = \frac{V_0}{d} = E_0$$

(*iii*) **Capacitance.** The *capacitance increases* from C_0 to C.

$$C = \kappa C_0$$

(*iv*) **Charge.** The *charge* on the capacitor plates *increases* from Q_0 to Q.

$$Q = CV = \kappa C_0 . V_0 = \kappa Q_0 .$$

(*v*) **Energy stored.** The *energy* stored in the capacitor *increases* κ times.

$$U = \frac{1}{2} CV^2 = \frac{1}{2} (\kappa C_0) V_0^2 = \kappa . \frac{1}{2} C_0 V_0^2 = \kappa U_0.$$

Table 2.2 Effect of dielectric on various parameters.

Battery disconnected from the capacitor	Battery kept connected across the capacitor
$Q = Q_0$ (constant)	$Q = \kappa Q_0$
$V = \frac{V_0}{\kappa}$	$V = V_0$ (constant)
$E = \frac{E_0}{\kappa}$	$E = E_0$ (constant)
$C = \kappa C_0$	$C = \kappa C_0$
$U = \frac{U_0}{\kappa}$	$U = \kappa U_0$

For Your Knowledge

➤ **Capacitance of a parallel plate capacitor with compound dielectric.**

A. Series type arrangement. If a capacitor is filled with n dielectric slabs of thicknesses $t_1, t_2, ..., t_n$, as shown in Fig. 2.122(*a*), then this arrangement is equivalent to n capacitors connected in series.

With a single dielectric slab,

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{\kappa}}$$

Capacitance with n dielectric slabs will be

$$C = \frac{\varepsilon_0 A}{d - (t_1 + t_2 + ... + t_n) + \left(\frac{t_1}{\kappa_1} + \frac{t_2}{\kappa_2} + ... + \frac{t_n}{\kappa_n} \right)}$$

But $d = t_1 + t_2 + t_3 + ... + t_n$

$$\therefore \quad C = \frac{\varepsilon_0 A}{\frac{t_1}{\kappa_1} + \frac{t_2}{\kappa_2} + ... + \frac{t_n}{\kappa_n}}$$

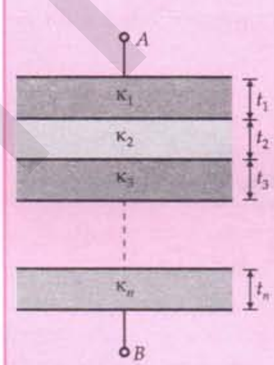


Fig. 2.122(*a*)

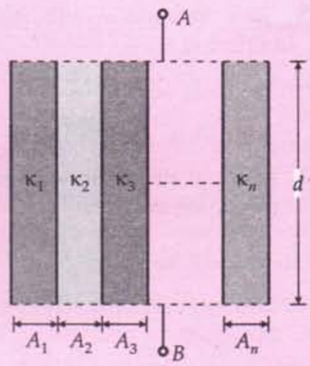


Fig. 2.122(*b*)

B. Parallel type arrangement. The arrangement shown in Fig. 2.122(*b*) consists of n capacitors in parallel, having plate areas $A_1, A_2, ..., A_n$, and plate separation d.

The equivalent capacitance of the parallel arrangement will be

$$C = C_1 + C_2 + ... + C_n$$

$$= \frac{\kappa_1 \varepsilon_0 A_1}{d} + \frac{\kappa_2 \varepsilon_0 A_2}{d} + ... + \frac{\kappa_n \varepsilon_0 A_n}{d}$$

or $\quad C = \frac{\varepsilon_0}{d} (\kappa_1 A_1 + \kappa_2 A_2 + ... + \kappa_n A_n)$

If $\quad A_1 = A_2 = ... = A_n = \frac{A}{n}$, then

$$C = \frac{\varepsilon_0 A}{d\, n} (\kappa_1 + \kappa_2 + ... + \kappa_n)$$

Examples based on Capacitors Filled with Dielectrics and Conductors

Formulae Used

1. Capacitance of a parallel plate capacitor filled with a dielectric of dielectric constant κ,
$$C = \kappa C_0 = \frac{\varepsilon_0 \kappa A}{d}$$
2. Capacitance of a parallel plate capacitor with a dielectric slab of thickness t ($< d$) in between its plates,
$$C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\kappa}\right)}$$
3. Capacitance of a parallel plate capacitor with a conducting slab of thickness t ($< d$) in between its plates,
$$C = \frac{\varepsilon_0 A}{d - t}$$
4. Capacitance of spherical capacitor filled with a dielectric,
$$C = 4\pi \varepsilon_0 \kappa . \frac{ab}{b-a}$$
5. Capacitance of a cylindrical capacitor filled with a dielectric,
$$C = \frac{2\pi \varepsilon_0 \kappa l}{2.303 \log_{10} \frac{b}{a}}$$
6. Effect of dielectric with battery disconnected from the capacitor,
$$Q = Q_0,\ V = \frac{V_0}{\kappa},\ E = \frac{E_0}{\kappa},\ C = \kappa C_0,\ U = \frac{U_0}{\kappa}$$
7. Effect of dielectric with battery connected across the capacitor,
$$Q = \kappa Q_0,\ V = V_0,\ E = E_0,\ C = \kappa C_0,\ U = \kappa U_0$$

Units Used

Capacitance C is in farad, charge q in coulomb, potential difference V in volt, area A in m^2, thicknesses d and t in metre.

Constant Used

Permittivity constant, $\varepsilon_0 = 8.85 \times 10^{-12} C^2N^{-1}m^{-2}$.

Example 79. *In a parallel plate capacitor, the capacitance increases from 4 μF to 80 μF, on introducing a dielectric medium between the plates. What is the dielectric constant of the medium ?*

Solution.
$$\kappa = \frac{\text{Capacitance with dielectric}}{\text{Capacitance without dielectric}} = \frac{80\ \mu F}{4\ \mu F} = \mathbf{20.}$$

Example 80. *A parallel plate capacitor with air between the plates has a capacitance of 8 μF. The separation between the plates is now reduced by half and the space between them is filled with a medium of dielectric constant 5. Calculate the value of capacitance of the capacitor in the second case.* [CBSE OD 06]

Solution. Capacitance of the capacitor with air between its plates,
$$C_0 = \frac{\varepsilon_0 A}{d} = 8 \text{ pF}$$
When the capacitor is filled with dielectric ($\kappa = 5$) between its plates and the distance between the plates is reduced by half, capacitance becomes
$$C = \frac{\varepsilon_0 \kappa A}{d/2} = \frac{\varepsilon_0 \times 5 \times A}{d/2} = 10\ C_0$$
or $$C = 10 \times 8 = \mathbf{80\ pF.}$$

Example 81. *Figure 2.123 shows two identical capacitors, C_1 and C_2, each of 1 μF capacitance connected to a battery of 6 V. Initially switch 'S' is closed. After some time 'S' is left open and dielectric slabs of dielectric constant $\kappa = 3$ are inserted to fill completely the space between the plates of the two capacitors. How will the (i) charge and (ii) potential difference between the plates of the capacitors be affected after the slabs are inserted ?* **[CBSE D 11]**

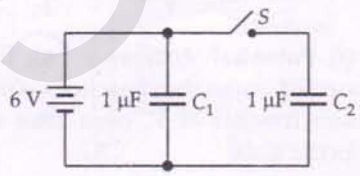


Fig. 2.123

Solution. With switch S closed, $V_1 = V_2 = 6$ V

$\therefore\ q_1 = q_2 = 1\ \mu F \times 6\ V = 6\ \mu C$

When dielectric slabs ($\kappa = 3$) are inserted, capacitance of each capacitor becomes 3 μF.

P.D. across C_1, $V_1' = \mathbf{6\ V}$

Charge, $q_1' = 3\ \mu F \times 6\ V = \mathbf{18\ \mu C}$

With switch S open, the p.d. on C_2 attains a new value but charge q_2 is still 6 μC.
$$\therefore \quad V_2' = \frac{6\ \mu C}{3\ \mu F} = \mathbf{2\ V.}$$

Example 82. *An ebonite plate ($\kappa = 3$), 6 mm thick, is introduced between the parallel plates of a capacitor of plate area $2 \times 10^{-2} m^2$ and plate separation 0.01 m. Find the capacitance.*

Solution. Here $t = 6$ mm $= 6 \times 10^{-3}$ m,
$$A = 2 \times 10^{-2}\ m^2,\quad d = 0.01\ m,\quad \kappa = 3$$
$$C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\kappa}\right)} = \frac{8.85 \times 10^{-12} \times 2 \times 10^{-2}}{0.01 - 6 \times 10^{-3}\left(1 - \frac{1}{3}\right)}$$
$$= \frac{17.7 \times 10^{-14}}{6 \times 10^{-3}} = 29.5 \times 10^{-12}\ F = \mathbf{29.5\ pF.}$$

Example 83. *Two parallel plate capacitors, X and Y, have the same area of plates and same separation between them. X has air between the plates while Y contains a dielectric medium of $\varepsilon_r = 4$.*

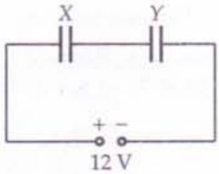


Fig. 2.124

(*i*) *Calculate capacitance of each capacitor if equivalent capacitance of the combination is 4 μF.*

(*ii*) *Calculate the potential difference between the plates of X and Y.*

(*iii*) *What is the ratio of electrostatic energy stored in X and Y ?* [CBSE D 04, 09]

Solution. (*i*) Let $C_X = C$. Then $C_Y = \varepsilon_r C = 4C$

Now X and Y are connected in series.

$$\therefore \quad C_{eq} = \frac{C_X C_Y}{C_X + C_Y} = \frac{C \cdot 4C}{C + 4C}$$

$$\text{or} \qquad 4\,\mu\text{F} = \frac{4}{5} C \quad \text{or} \quad C = 5\,\mu\text{F}$$

Hence $C_X = C = \mathbf{5\,\mu F}$ and $C_Y = 4C = 4 \times 5 = \mathbf{20\,\mu F}$.

(*ii*) Let V be the p.d. across X. Then p.d. across Y will be $V/4$.

$$\therefore \quad V + \frac{V}{4} = 12 \quad \text{or} \quad V = 9.6\text{ V}$$

Hence $V_X = V = \mathbf{9.6\ V}$ and $V_Y = V/4 = \mathbf{2.4\ V}$.

$$(iii)\ \frac{\text{Energy stored in } X}{\text{Energy stored in } Y} = \frac{\frac{1}{2}C(9.6)^2}{\frac{1}{2}4C(2.4)^2} = \frac{4}{1} = \mathbf{4:1}.$$

Example 84. *An electric field $E_0 = 3 \times 10^4\ Vm^{-1}$ is established between the plates, 0.05 m apart, of a parallel plate capacitor. After removing the charging battery, an uncharged metal plate of thickness $t = 0.01$ m is inserted between the capacitor plates. Find the p.d. across the capacitor (i) before, (ii) after the introduction of the plate. (iii) What would be the p.d. if a dielectric slab ($\kappa = 2$) were introduced in place of metal plate ?* [Roorkee 91]

Solution. (*i*) The p.d. across the capacitor plates before metal plate is inserted,

$$V_0 = E_0 d = 3 \times 10^4 \times 0.05 = \mathbf{1500\ V}.$$

(*ii*) As no electric field exists in metal plate, so the p.d. after the introduction of metal plate is

$$V = E_0 (d - t) = 3 \times 10^4 \times (0.05 - 0.01) = \mathbf{1200\ V}.$$

(*iii*) When dielectric slab ($\kappa = 2$) is introduced, the p.d. becomes

$$V = E_0(d - t) + \frac{E_0}{\kappa} t = 1200 + \frac{3 \times 10^4 \times 0.01}{2} = \mathbf{1350\ V}.$$

Example 85. *A parallel plate capacitor is charged to a certain potential difference. When a 3.0 mm thick slab is slipped between the capacitor plates, then to maintain the same p.d. between the plates, the plate separation is to be increased by 2.4 mm. Find the dielectric constant of the slab.*

Solution. Let E_0 be the electric field between the capacitor plates before the introduction of the slab. Then, the p.d. between the plates is

$$V_0 = E_0 d$$

Suppose the separation between the plates is increased by d' to maintain the same p.d. after the introduction of the slab of thickness t. Then

$$V_0 = E_0 (d + d' - t) + \frac{E_0}{\kappa} \cdot t$$

$$\therefore \quad E_0 (d + d' - t) + \frac{E_0}{\kappa} \cdot t = E_0 d$$

$$\text{or} \qquad \kappa = \frac{t}{t - d'} = \frac{3.0\text{ mm}}{3.0\text{ mm} - 2.4\text{ mm}} = \mathbf{5}.$$

Example 86. *The area of parallel plates of an air-filled capacitor is 0.20 m^2 and the distance between them is 0.01 m. The p.d. across the plates is 3000 V. When a 0.01 m thick dielectric sheet is placed between the plates, the p.d. decreases to 1000 V. Determine (i) capacitance of the capacitor before placing the sheet (ii) charge on each plate (iii) dielectric constant of the material (iv) capacitance of the capacitor after placing the dielectric (v) permittivity of the dielectric. Given $\varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$.*

Solution. (*i*) Capacitance of air-filled capacitor is

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 0.20}{0.01} = \mathbf{1.77 \times 10^{-10}\ F}.$$

(*ii*) Charge on each plate,

$$q = C_0 V_0 = 1.77 \times 10^{-10} \times 3000 = \mathbf{5.31 \times 10^{-7}\ C}.$$

(*iii*) Dielectric constant of the material is

$$\kappa = \frac{C}{C_0} = \frac{q/V}{q/V_0} = \frac{V_0}{V} = \frac{3000}{1000} = \mathbf{3}.$$

(*iv*) Capacitance after the dielectric sheet is introduced,

$$C = \kappa C_0 = 3 \times 1.77 \times 10^{-10} = \mathbf{5.31 \times 10^{-10}\ F}.$$

(*v*) Permittivity of the dielectric is

$$\varepsilon = \kappa \varepsilon_0 = 3 \times 8.85 \times 10^{-12} = \mathbf{2.65 \times 10^{-11}\ Fm^{-1}}.$$

Example 87. *The capacitance of a parallel plate capacitor is 50 pF and the distance between the plates is 4 mm. It is charged to 200 V and then the charging battery is removed. Now a dielectric slab ($\kappa = 4$) of thickness 2 mm is placed. Determine (i) final charge on each plate (ii) final potential difference between the plates (iii) final energy in the capacitor and (iv) energy loss.*

Solution. Capacitance of air-filled capacitor,

$$C_0 = \frac{\varepsilon_0 A}{d} \qquad \text{...(1)}$$

Capacitance with dielectric slab of thickness $t\,(< d)$ is

$$C = \frac{\varepsilon_0 A}{d - t + t/\kappa} \qquad ...(2)$$

(*i*) The charge on capacitor plates, when 200 V p.d. is applied, becomes

$$q = C_0 V_0 = 50 \times 10^{-12} \times 200 = 10^{-8}\ \text{C}$$

Even after the battery is removed, the charge of 10^{-8} C on the capacitor plates remains the same.

(*ii*) On placing the dielectric slab, suppose the capacitance becomes C and potential difference V. Then

$$q = C_0 V_0 = CV$$

or
$$V = \frac{C_0}{C} V_0 = \frac{d - t + t/\kappa}{d} V_0$$

[Using (1) and (2)]

$$= \frac{4 - 2 + 2/4}{4} \times 200 = \mathbf{125\ V.}$$

(*iii*) Final energy in the capacitor is

$$U = \frac{1}{2} qV = \frac{1}{2} \times 10^{-8} \times 125 = \mathbf{6.25 \times 10^{-7}\ J.}$$

(*iv*) Energy loss

$$= U_0 - U = \frac{1}{2} q (V_0 - V)$$

$$= \frac{1}{2} \times 10^{-8} \times (200 - 125)$$

$$= \mathbf{3.75 \times 10^{-7}\ J.}$$

Example 88. *A parallel plate capacitor is formed by two plates, each of area 100 cm^2, separated by a distance of 1 mm. A dielectric of dielectric constant 5 and dielectric strength $1.9 \times 10^7\ Vm^{-1}$ is filled between the plates. Find the maximum charge that can be stored on the capacitor without causing any dielectric breakdown.*

Solution. Electric field between capacitor plates is given by

$$E = \frac{\sigma}{\kappa\, \varepsilon_0} = \frac{q}{\kappa\, \varepsilon_0\, A}$$

As the electric field should not exceed $1.9 \times 10^7\ \text{Vm}^{-1}$, so the maximum charge that can be stored is

$$q = \kappa\, \varepsilon_0\, AE$$

$$= 5 \times 8.85 \times 10^{-12} \times 100 \times 10^{-4} \times 1.9 \times 10^7$$

$$= \mathbf{8.4 \times 10^{-6}\ C.}$$

Example 89. *A slab of material of dielectric constant κ has the same area as the plates of a parallel plate capacitor but has a thickness 3d / 4, where d is the separation of the plates. How is the capacitance changed when the slab is inserted between the plates?* [NCERT]

Solution. If V_0 is the potential difference when there is no dielectric, then the electric field between the capacitor plates will be

$$E_0 = \frac{V_0}{d}$$

After the dielectric is inserted, the electric field in the dielectric reduces to

$$E = \frac{E_0}{\kappa}$$

Now the potential difference between the plates will be

$$V = E_0 \cdot \frac{d}{4} + E \cdot \frac{3d}{4} = E_0 \cdot \frac{d}{4} + \frac{E_0}{\kappa} \cdot \frac{3d}{4}$$

$$= E_0 d \left(\frac{1}{4} + \frac{3}{4\kappa} \right) = V_0 \frac{\kappa + 3}{4\kappa}$$

Thus the potential difference decreases by a factor of $(\kappa + 3)/4\kappa$, while the free charge q_0 on the plates remains same. The capacitance increases to a new value given by

$$C = \frac{q_0}{V} = \frac{4\kappa}{\kappa + 3} \cdot \frac{q_0}{V_0} = \frac{4\kappa}{\kappa + 3} C_0.$$

Example 90

(*a*) *Find the ratio of the capacitances of a capacitor filled with two dielectrics of same dimensions but of dielectric constants κ_1 and κ_2, respectively.*

(*b*) *A capacitor is filled with two dielectrics of the same dimensions but of dielectric constants $\kappa_1 = 2$ and $\kappa_2 = 3$. Find the ratio of capacities in two possible arrangements.* [MNREC 85]

Solution. (*a*) The two possible arrangements of the two dielectrics are shown in Figs. 2.125(*a*) and (*b*).

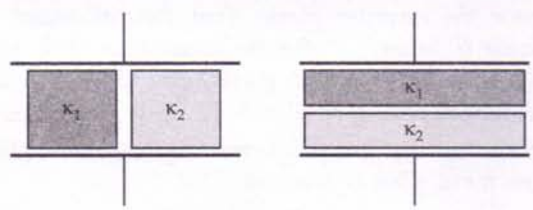


Fig. 2.125 (*a*) (*b*)

(*i*) The arrangement (*a*) can be supposed to be a parallel combination of two capacitors, each with plate area *A*/2 and separation *d*. Therefore, the total capacitance is

$$C = C_1 + C_2 = \frac{\varepsilon_0 (A/2)\, \kappa_1}{d} + \frac{\varepsilon_0 (A/2)\, \kappa_2}{d}$$

$$= \frac{\varepsilon_0\, A(\kappa_1 + \kappa_2)}{2d}.$$

(*ii*) The arrangement (*b*) can be supposed to be a series combination of two capacitors, each with plate area A and separation $d/2$. Therefore, the total capacitance C' is given by

$$\frac{1}{C'}=\frac{1}{C_1'}+\frac{1}{C_2'}=\frac{1}{\frac{\varepsilon_0 A\kappa_1}{d/2}}+\frac{1}{\frac{\varepsilon_0 A\kappa_2}{d/2}}$$

$$=\frac{d}{2\varepsilon_0 A}\left(\frac{1}{\kappa_1}+\frac{1}{\kappa_2}\right)$$

or $$C'=\frac{2\varepsilon_0 A}{d}\left(\frac{\kappa_1\kappa_2}{\kappa_1+\kappa_2}\right)$$

Ratio of the capacitances in the two arrangements is

$$\frac{C}{C'}=\frac{\varepsilon_0 A(\kappa_1+\kappa_2)}{2d}\cdot\frac{d(\kappa_1+\kappa_2)}{2\varepsilon_0 A\kappa_1\kappa_2}=\frac{(\kappa_1+\kappa_2)^2}{4\kappa_1\kappa_2}.$$

(*b*) Here $\kappa_1=2, \kappa_2=3$

$$\therefore \quad \frac{C}{C'}=\frac{(2+3)^2}{4\times2\times3}=\frac{25}{24}.$$

Problems For Practice

1. A parallel-plate capacitor having plate area 100 cm^2 and separation 1.0 mm holds a charge of 0.12 μC when connected to a 120 V battery. Find the dielectric constant of the material filling the gap. (Ans. 11.3)

2. Find the length of the paper used in a capacitor of capacitance 2 μF, if the dielectric constant of the paper is 2.5 and its width and thickness are 50 mm and 0.05 mm, respectively. (Ans. 90 m)

3. A parallel-plate capacitor consists of 26 metal strips, each of 3 cm × 4 cm, separated by mica sheets of dielectric constant 6 and uniform thickness 0.2 mm. Find the capacitance. (Ans. 7.97×10^{-9} F)

4. A parallel-plate capacitor of capacity 0.5 μF is to be constructed using paper sheets of thickness 0.04 mm as dielectric. Find how many circular metal foils of diameter 0.1 m will have to be used. Take the dielectric constant of paper used as 4. (Ans. 73)

5. When a slab of insulating material 4 mm thick is introduced between the plates of a parallel plate capacitor, it is found that the distance between the plates has to be increased by 3.2 mm to restore the capacitance to the original value. Calculate the dielectric constant of the material. (Ans. 5)

6. The two plates of a parallel plate capacitor are 4 mm apart. A slab of dielectric constant 3 and thickness 3 mm is introduced between the plates with its faces parallel to them. The distance between the plates is so adjusted that the capacitance of the capacitor becomes 2/3rd of its original value. What is the new distance between the plates ? [CBSE OD 08C] (Ans. 8 mm)

7. The distance between the parallel plates of a charged capacitor is 5 cm and the intensity of electric field is 300 V cm^{-1}. A slab of dielectric constant 5 and thickness 1 cm is inserted parallel to the plates. Determine the potential difference between the plates, before and after the slab is inserted ? (Ans. 1500 V, 1260 V)

8. A parallel plate capacitor with plate separation 5 mm is charged by a battery. It is found that on introducing a mica sheet 2 mm thick, while keeping the battery connections intact, the capacitor draws 25% more energy from the battery than before. Find the dielectric constant of mica. (Ans. 2)

9. Figure 2.126 shows a parallel plate capacitor of plate area A and plate separation d. Its entire space is filled with three different dielectric slabs of same thickness. Find the equivalent capacitance of the arrangement. $\left[\text{Ans. } C=\frac{3\varepsilon_0 A\kappa_1\kappa_2\kappa_3}{d(\kappa_1\kappa_2+\kappa_2\kappa_3+\kappa_3\kappa_1)}\right]$

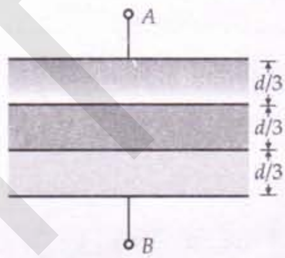


Fig. 2.126

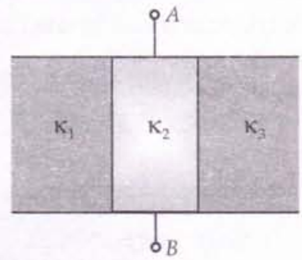


Fig. 2.127

10. The space between the plates of a parallel plate capacitor of capacitance C is filled with three dielectric slabs of equal thickness, as shown in Fig. 2.127. If the dielectric constants of the three slabs are κ_1, κ_2 and κ_3, find the new capacitance. $\left[\text{Ans. } C'=\frac{C}{3}(\kappa_1+\kappa_2+\kappa_3)\right]$

11. A slab of material of dielectric constant κ has the same area as the plates of a parallel plate capacitor but has thickness $d/2$, where d is the separation between the plates. Find the expression for the capacitance when the slab is inserted between the plates. [CBSE F 10 ; OD 13] $\left(\text{Ans. } \frac{2\kappa}{\kappa+1}C_0\right)$

HINTS

1. Capacitance, $C=\frac{q}{V}=\frac{0.12\,\mu C}{120\text{ V}}=10^{-9}$ F

If κ is the dielectric constant, then

$$C=\frac{\kappa\varepsilon_0 A}{d}=\frac{\kappa\times8.85\times10^{-12}\times100\times10^{-4}}{1.0\times10^{-3}}=10^{-9}\text{F}$$

$\therefore \quad \kappa=$ **11.3**.

3. Arrangement of n metal plates separated by dielectric acts as a parallel combination of $(n-1)$ capacitors.

$$\therefore \quad C = \frac{(n-1)\kappa \varepsilon_0 A}{d}$$
$$= \frac{25 \times 6 \times 8.85 \times 10^{-12} \times 3 \times 4 \times 10^{-4}}{0.2 \times 10^{-3}}$$
$$= \mathbf{7.97 \times 10^{-9}\ F.}$$

4. As $C = \frac{(n-1)\kappa \varepsilon_0 A}{d}$

$$\therefore \ 0.5 \times 10^{-6}\text{ F} = \frac{(n-1) \times 4 \times 8.85 \times 10^{-12} \times 3.14 \times (0.05)^2}{0.04 \times 10^{-3}}$$

or $$n - 1 = \frac{0.5 \times 0.04 \times 10^3}{4 \times 8.85 \times 3.14 \times (0.05)^2} = 71.97 \simeq 72$$

or $n = \mathbf{73}$.

5. Capacitance without dielectric, $C = \frac{\varepsilon_0 A}{d}$

When dielectric is introduced, $C = \frac{\varepsilon_0 A}{d' - t\left(1 - \frac{1}{\kappa}\right)}$

As the capacitance remains same in both cases, so

$$\frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d' - t\left(1 - \frac{1}{\kappa}\right)}$$

or $$d = d' - t\left(1 - \frac{1}{\kappa}\right) \quad \text{or} \quad d' - d = t\left(1 - \frac{1}{\kappa}\right)$$

But $d' - d = 3.2$ mm, $t = 4$ mm

$$\therefore \quad 3.2 = 4\left(1 - \frac{1}{\kappa}\right)$$

or $$1 - \frac{1}{\kappa} = \frac{3.2}{4} = 0.8 \quad \text{or} \quad \frac{1}{\kappa} = 0.2 \quad \text{or} \quad \kappa = \mathbf{5}.$$

6. $\frac{2}{3} \times$ Capacitance with air = Capacitance with dielectric

or $$\frac{2}{3}\frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d' - t + \frac{t}{\kappa}}$$

or $$\frac{2}{3}\left(d' - 3 + \frac{3}{3}\right) = d = 4\text{ mm} \quad \text{or} \quad d' = \mathbf{8\ mm}$$

7. P.D. before the dielectric slab is inserted,

$$V_0 = E_0 d = 300\text{ V cm}^{-1} \times 5\text{ cm} = \mathbf{1500\ V}.$$
$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{0.05}\text{ farad}$$

or $\varepsilon_0 A = 0.05\, C_0$

Capacitance with dielectric slab,

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{\kappa}} = \frac{0.05\, C_0}{0.05 - 0.01 + \frac{0.01}{5}} = \frac{25\, C_0}{21}$$

For charge to remain constant,

$$C_0 V_0 = CV$$
$$C_0 \times 1500 = \frac{25\, C_0}{21} \times V \quad \text{or} \quad V = \mathbf{1260\ V.}$$

8. As the battery connections are intact (V = constant) and the capacitor draws 25% more charge, so the capacitance also increases by 25%. That is

$$C = \frac{125}{100} C_0 = \frac{5}{4} C_0$$

or $$\frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\kappa}\right)} = \frac{5}{4} \cdot \frac{\varepsilon_0 A}{d}$$

or $$d - t\left(1 - \frac{1}{\kappa}\right) = \frac{4d}{5} \quad \text{or} \quad t\left(1 - \frac{1}{\kappa}\right) = \frac{d}{5}$$

or $$1 - \frac{1}{\kappa} = \frac{d}{5t} = \frac{5}{5 \times 2} = \frac{1}{2} \quad \text{or} \quad \kappa = \mathbf{2}$$

9. The given arrangement is equivalent to three capacitors connected in series. Each such capacitor has plate area A and plate separation d.

$$\therefore \quad C_1 = \frac{\kappa_1 \varepsilon_0 A}{d/3} = \frac{3\kappa_1 \varepsilon_0 A}{d}$$
$$C_2 = \frac{3\kappa_2 \varepsilon_0 A}{d} \quad \text{and} \quad C_3 = \frac{3\kappa_3 \varepsilon_0 A}{d}$$

The equivalent capacity C of the given arrangement is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{d}{3\varepsilon_0 A}\left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2} + \frac{1}{\kappa_3}\right)$$

or $$C = \frac{3\varepsilon_0 A \kappa_1 \kappa_2 \kappa_3}{d(\kappa_1 \kappa_2 + \kappa_2 \kappa_3 + \kappa_3 \kappa_1)}.$$

10. Original capacitance, $C = \frac{\varepsilon_0 A}{d}$

The new arrangement is equivalent to three capacitors connected in parallel. Each such capacitor has plate area $A/3$ and plate separation d. The new capacitance is

$$C' = C_1 + C_2 + C_3$$
$$= \frac{\kappa_1 \varepsilon_0 A/3}{d} + \frac{\kappa_2 \varepsilon_0 A/3}{d} + \frac{\kappa_3 \varepsilon_0 A/3}{d}$$
$$= \frac{\varepsilon_0 A}{3d}(\kappa_1 + \kappa_2 + \kappa_3)$$

or $$C' = \frac{C}{3}(\kappa_1 + \kappa_2 + \kappa_3).$$

11. Without dielectric, $E_0 = \frac{V_0}{d}$

With dielectric, $E = \frac{E_0}{\kappa}$

$$\therefore \quad V = E_0 \cdot \frac{d}{2} + E \cdot \frac{d}{2} = E_0 \cdot \frac{d}{2} + \frac{E_0}{\kappa} \cdot \frac{d}{2}$$
$$= \frac{E_0 d}{2} \cdot \frac{(\kappa + 1)}{\kappa} = \frac{V_0(\kappa + 1)}{2\kappa}$$

or $$C = \frac{q_0}{V} = \frac{2\kappa q_0}{V_0(\kappa + 1)} = \frac{2\kappa}{\kappa + 1} C_0$$

2.33 DISCHARGING ACTION OF SHARP POINTS : CORONA DISCHARGE

54. *Briefly explain discharging action of sharp points or corona discharge.*

Discharging action of sharp points : Corona discharge. When a spherical conductor of radius r carries a charge q, its surface charge density is

$$\sigma = \frac{q}{A} = \frac{q}{4\pi r^2}$$

Electric field on the surface is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{4\pi\varepsilon_0 r^2}$$

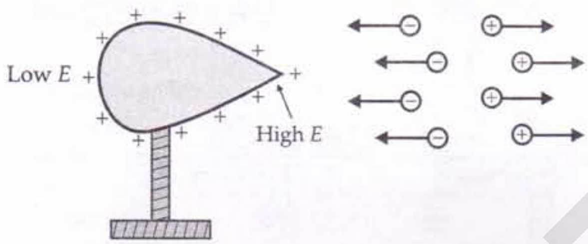


Fig. 2.128 Corona discharge.

The pointed end of a conductor is highly curved and its radius of curvature r very small. If the conductor is given a charge q, then the charge density σ at the pointed end will be very high. Consequently, the electric field near the pointed end will be very high which may cause the ionisation or electrical breakdown of the surrounding air. The oppositely charged ions neutralise the pointed end while the similarly charged ions are repelled away. Fresh air molecules come near the pointed end and take away its charge, setting up a kind of *electric wind. This process by which the charge at the pointed end of a conductor gets discharged is called* ***corona discharge****.* The discharge is often accompanied by a visible glow near the pointed end.

2.34 COLLECTING ACTION OF A HOLLOW CONDUCTOR

55. *A small sphere of radius r and charge q is enclosed by a spherical shell of radius R and charge Q. Show that if q is positive, charge q will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge Q on the shell is.* [NCERT]

Collecting action of a hollow sphere. Consider a small sphere of radius r placed inside a large spherical shell of radius R. Let the spheres carry charges q and Q, respectively.

Total potential on the outer sphere,

V_R = Potential due to its own charge Q
+ Potential due to the charge q on the inner sphere

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{R} + \frac{q}{R}\right]$$

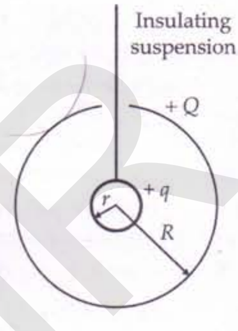


Fig. 2.129 Small charged sphere suspended inside a charged spherical shell.

Potential on the inner sphere due to its own charge is

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

As the potential at every point inside a charged sphere is the same as that on its surface, so potential on the inner sphere due to charge Q on outer sphere is

$$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R}$$

$\therefore$ Total potential on inner sphere

$$V_r = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r} + \frac{Q}{R}\right]$$

Hence the potential difference is

$$V_r - V_R = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{R}\right]$$

So if q is positive, the potential of the inner sphere will always be higher than that of the outer sphere. Now if the two spheres are connected by a conducting wire, the charge q will flow entirely to the outer sphere, irrespective of the charge Q already present on the outer sphere. In fact this is true for conductors of any shape.

2.35 VAN DE GRAAFF GENERATOR*

56. *Explain the basic principle, construction and working of Van de Graaff generator.*

Van de Graaff generator. *It is an electrostatic generator capable of building up high potential differences of the order of* 10^7 *volts.*

Principle. The working of a Van de Graaff generator is based on following two electrostatic phenomena :

(*i*) *Discharging action of sharp points (corona discharge) i.e., electric discharge takes place in air or gases readily at the pointed ends of conductors.*

(*ii*) *If a charged conductor is brought into internal contact with a hollow conductor, all of its charge transfers to the hollow conductor, howsoever high the potential of the latter may be.*

Construction. A large spherical conducting shell (of few metres radius) is supported at a height several metres above the ground on an insulating column. A long narrow belt of insulating material, like rubber or silk, is wound around two pulleys, P_1 at ground level and P_2 at the centre of the shell.

This belt is kept continuously moving by an electric motor attached to the lower pulley P_1. Near the bottom and the top of its run, the belt passes close to two sharply pointed brass combs B_1 and B_2, pointing towards the belt. The comb B_1, called *spray comb* is given a positive potential of 10 kV with respect to the earth by means of a battery ; while the comb B_2, called *collecting comb*, is connected to the spherical shell S.

Working. Due to the high electric field at the pointed ends of comb B_1, the air of the neighbourhood gets ionised and its positive charge repelled or sprayed on to the belt, which moves up into the shell S. As it passes close to comb B_2, it induces a negative charge at the pointed ends of comb B_2 and a positive charge on the shell S. The positive charge spreads uniformly on the outer surface of the shell S. The high electric field at the pointed ends of comb B_2 ionises the air there and repels the negative charges on to the belt which neutralise its positive charge. This process continues. As more and more positive charge is given to the shell, its potential continues to rise. In this way, a high potential of 6 to 8 million volts can be built upon the sphere.

A discharge tube is placed with its upper end inside the hollow sphere and lower end earthed. The ion source is placed at the upper end of the tube. The high potential on the sphere repels the charged particles downward with large acceleration, where they hit the target atoms to bring about the nuclear disintegration.

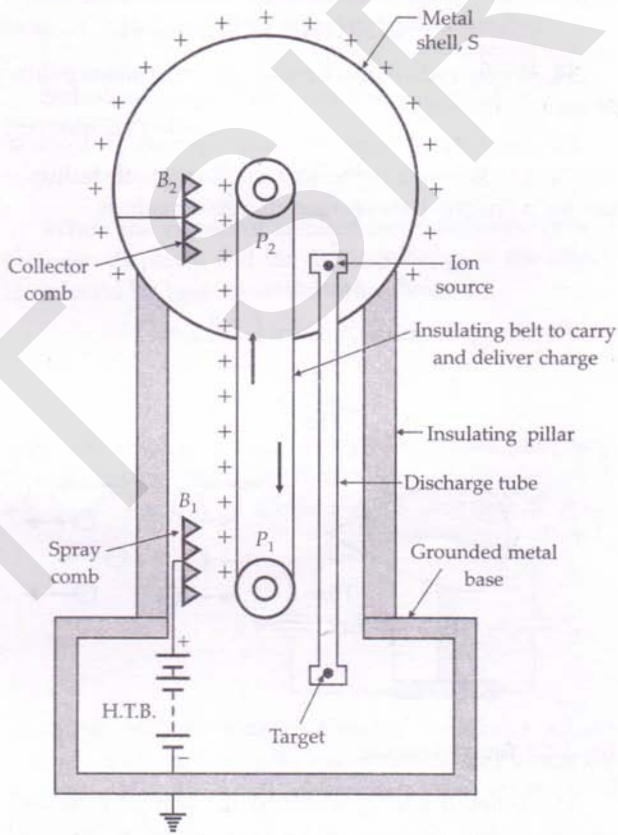


Fig. 2.130

Use. The high potential difference set up in a Van de Graaff generator is used to accelerate charged particles like protons, deutrons, α-particles, etc. to high energies of about 10 MeV, needed for experiments to probe the small scale structure of matter.

2.1. *Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point on the line joining the two charges is the electric potential zero ? Take the potential at infinity to be zero.*

Ans. Zero of electric potential for two charges. As shown in Fig. 2.186, suppose the two charges are placed on X-axis with the positive charge located at the origin O.

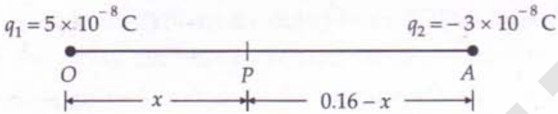


Fig. 2.186

Let the potential be zero at the point P and $OP = x$. For $x < 0$ (*i.e.*, to the left of O), the potentials of the two charges cannot add up to zero. Clearly, x must be positive. If x lies between O and A, then

$$V_1 + V_2 = 0$$

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{x} + \frac{q_2}{0.16 - x}\right] = 0$$

or $$9 \times 10^9 \left[\frac{5 \times 10^{-8}}{x} - \frac{3 \times 10^{-8}}{0.16 - x}\right] = 0$$

or $$\frac{5}{x} - \frac{3}{0.16 - x} = 0$$

or $$x = 0.10 \text{ m} = \mathbf{10 \text{ cm}}.$$

The other possibility is that x may also lie on OA produced, as shown in Fig. 2.187.

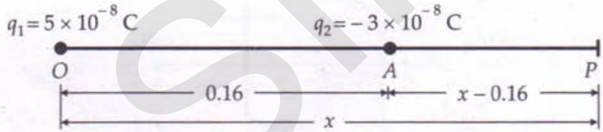


Fig. 2.187

As $$V_1 + V_2 = 0$$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\left[\frac{5 \times 10^{-8}}{x} - \frac{3 \times 10^{-8}}{x - 0.16}\right] = 0$$

or $$\frac{5}{x} - \frac{3}{x - 0.16} = 0$$

or $$x = 0.40 \text{ m} = \mathbf{40 \text{ cm}}.$$

Thus the electric potential is zero at 10 cm and 40 cm away from the positive charge on the side of the negative charge.

2.2. *A regular hexagon of side 10 cm has a charge of 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon.*

Ans. Clearly, distance of each charge from the centre O is

$$r = 10 \text{ cm} = 0.10 \text{ m}$$

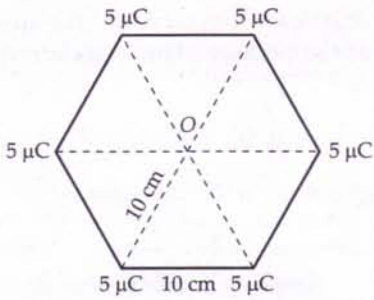


Fig. 2.188

Magnitude of each charge is

$$q = 5\,\mu C = 5\times10^{-6} C$$

∴ Potential at the centre O is

$$V = 6\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} = \frac{6\times9\times10^9\times5\times10^{-6}}{0.10}$$

$$= \mathbf{2.7\times10^6\ V.}$$

2.3. *Two charges $+2\,\mu C$ and $-2\,\mu C$ are placed at points A and B, 6 cm apart. (i) Identify an equipotential surface of the system (ii) What is the direction of the electric field at every point on the surface ?*

Ans. (*i*) The equipotential surface will be a plane normal to AB and passing through its midpoint O, as shown in Fig. 2.189. It has zero potential everywhere.

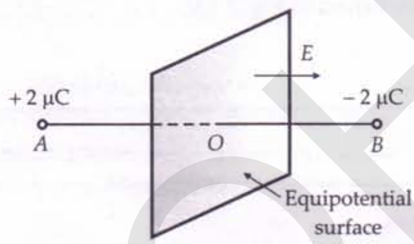


Fig. 2.189

(*ii*) The direction of electric field is normal to the plane in the direction AB *i.e.*, from positive to negative charge.

2.4. *A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7} C distributed uniformly on its surface. What is the electric field*

(a) inside the sphere (b) just outside the sphere

(c) at a point 18 cm from the centre of the sphere ?

Ans. Refer to the solution of Example 75 on page 1.60.

2.5. *A parallel plate capacitor with air between the plates has a capacitance of 8 pF ($1\,pF = 10^{-12}$ F). What will be the capacitance if the distance between the plates be reduced by half, the space between them is filled with a substance of dielectric constant, $\kappa = 6$?*

Ans. Capacitance of the capacitor with air between its plates,

$$= \frac{\varepsilon_0 A}{d} = 8\text{ pF}$$

When the capacitor is filled with dielectric ($\kappa = 6$) between its plates and the distance between the plates is reduced by half, capacitance becomes,

$$C' = \frac{\varepsilon_0\kappa A}{d'} = \frac{\varepsilon_0\times6\times A}{d/2} = 12\frac{\varepsilon_0 A}{d}$$

or $$C' = 12\times8 = \mathbf{96\ pF}.$$

2.6. *Three capacitors each of capacitance 9 pF are connected in series. (a) What is the total capacitance of the combination ? (b) What is the potential difference across each capacitor when the combination is connected to a 120 V supply ?*

Ans. (*a*) If C is the equivalent capacitance of the series combination, then

$$\frac{1}{C} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3} = \frac{1}{9}+\frac{1}{9}+\frac{1}{9} = \frac{3}{9} = \frac{1}{3}$$

or $$C = \mathbf{3\ pF}.$$

(*b*) As all the capacitors have equal capacitance, so potential drop ΔV would be same across each capacitor.

$$\therefore\quad V = \Delta V_1 + \Delta V_2 + \Delta V_3$$

$$= \Delta V + \Delta V + \Delta V = 3\Delta V$$

or $$\Delta V = \frac{V}{3} = \frac{120}{3} = \mathbf{40\ V.}$$

2.7. *Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel. (a) What is the total capacitance of the combination ? (b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.*

Ans. (*a*) For the parallel combination, total capacitance is given by

$$C = C_1 + C_2 + C_2 = 2+3+4 = \mathbf{9\ pF.}$$

(*b*) When the combination is connected to 100 V supply, charges on the capacitors will be

$$q_1 = C_1V = 2\times10^{-12}\times100 = \mathbf{2\times10^{-10}\,C}$$

$$q_2 = C_2V = 3\times10^{-12}\times100 = \mathbf{3\times10^{-10}\,C}$$

$$q_2 = C_3V = 4\times10^{-12}\times100 = \mathbf{4\times10^{-10}\,C.}$$

2.8. *In a parallel plate capacitor with air between the plates, each plate has an area of $6\times10^{-3}\,m^2$ and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If the capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor ?*

Ans. Capacitance of capacitor with air between its plates is

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\times6\times10^{-3}}{3\times10^{-3}}$$

$$= 1.8\times10^{-11}\,F = \mathbf{18\ pF.}$$

Charge,

$$q = C_0V = 1.8\times10^{-11}\times100 = \mathbf{1.8\times10^{-9}\ C.}$$

2.9. *Explain what would happen if in the capacitor given in Exercise 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates, (i) while the voltage supply remains connected (ii) after the supply was disconnected.*

[CBSE Sample Paper 98]

Ans. From the above question, we have

$$C_0 = 1.8 \times 10^{-11} \text{ F} = 18 \text{ pF}, \quad q_0 = 1.8 \times 10^{-9} \text{ C}$$

Also, $\kappa = 6$

(*i*) *When the voltage supply remains connected*, the potential difference between capacitor plates remains same *i.e.*, **100 V**.

The capacitance increases κ times.

$$\therefore \quad C = \kappa C_0 = 6 \times 18 = \mathbf{108 \text{ pF}}.$$

The charge on the capacitor plates will be

$$q = CV = 108 \times 10^{-12} \times 100 = \mathbf{1.08 \times 10^{-8} \text{ C}}.$$

(*ii*) *After the supply is disconnected*, the charge on the capacitor plates remains same *i.e.*,

$$q_0 = 1.8 \times 10^{-9} \text{ C}$$

The capacitance increases κ times.

$$C = \kappa C_0 = \mathbf{108 \text{ pF}}.$$

The potential difference between the capacitor plates becomes

$$V = \frac{V_0}{\kappa} = \frac{100}{6} = \mathbf{16.6 \text{ V}}.$$

2.10. *A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor ?*

Ans. Here $C = 12 \text{ pF} = 12 \times 10^{-12}$ F, $V = 50$ V

Energy stored,

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times 12 \times 10^{-12} \times (50)^2 = \mathbf{1.5 \times 10^{-8} \text{ J}}.$$

2.11. *A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process ?*

Ans. Here $C_1 = 600$ pF, $V_1 = 200$ V,

$C_2 = 600$ pF, $V_2 = 0$

Common potential,

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{600 \times 10^{-12} \times 200 + 0}{(600 + 600) \times 10^{-12}} = 100 \text{ V}$$

Initial energy stored,

$$U_i = U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times 600 \times 10^{-12} \times (200)^2$$

$$= 12 \times 10^{-6} \text{ J}$$

Final energy stored,

$$U_f = \frac{1}{2}(C_1 + C_2) V^2$$

$$= \frac{1}{2}(600 + 600) \times 10^{-12} \times (100)^2 = 6 \times 10^{-6} \text{ J}$$

Electrostatic energy lost,

$$\Delta U = U_i - U_f = 12 \times 10^{-6} - 6 \times 10^{-6}$$

$$= \mathbf{6 \times 10^{-6} \text{ J}}$$

2.12. *A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of* -2×10^{-9} *C from a point P (0, 0, 3 cm) to a point Q (0, 4 cm, 0) via a point R (0, 6 cm, 9 cm).*

Ans. As the work done in taking a charge from one point to another is independent of the path followed, therefore

$$W = q_0 [V_Q - V_P] = q_0 \left[\frac{q}{4\pi \varepsilon_0 r_2} - \frac{q}{4\pi \varepsilon_0 r_1} \right]$$

$$= \frac{q_0 q}{4\pi \varepsilon_0} \left[\frac{1}{r_2} - \frac{1}{r_1} \right]$$

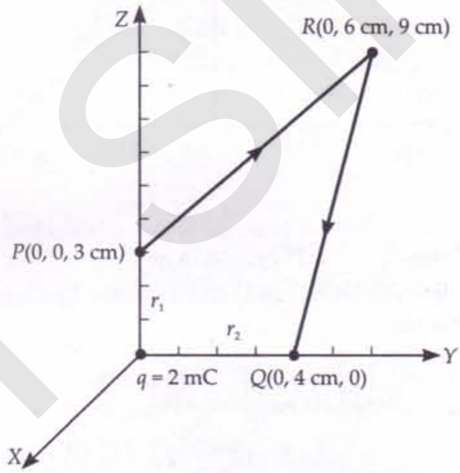


Fig. 2.190

Here $q = 8 \text{ mC} = 8 \times 10^{-3}\text{C}, q_0 = -2 \times 10^{-9}$ C

$r_1 = 3 \text{ cm} = 3 \times 10^{-2}$ m, $r_2 = 4 \text{ cm} = 4 \times 10^{-2}$ m

$$\therefore \quad W = -2 \times 10^{-9} \times 8 \times 10^{-3} \times 9 \times 10^{9} \times \left[\frac{1}{4 \times 10^{-2}} - \frac{1}{3 \times 10^{-2}} \right]$$

$$= \mathbf{1.2 \text{ J}}.$$

2.13. *A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.*

Ans. Length of longest diagonal of the cube

$$= \sqrt{b^2 + b^2 + b^2} = \sqrt{3}\, b$$

Distance of each charge (placed at vertex) from the centre of the cube is

$$r = \frac{\sqrt{3}}{2} b$$

$\therefore$ Potential at the centre of the cube is

$$V = 8 \cdot \frac{1}{4\pi \varepsilon_0} \cdot \frac{q}{r} = 8 \times \frac{1}{4\pi \varepsilon_0} \cdot \frac{2q}{\sqrt{3}\, b}$$

$$= \frac{\mathbf{4q}}{\mathbf{\sqrt{3}\, \pi \varepsilon_0 b}}.$$

Electric fields at the centre due to any pair of charges at the opposite corners will be equal and opposite thus cancelling out in pairs. Hence resultant electric field at the centre will be zero.

2.14. *Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential (a) at the midpoint of the line joining the two charges, and (b) at a point 10 cm from this midpoint in a plane normal to the line and passing through the midpoint.*

Ans. The two situations are shown in Fig. 2.191.

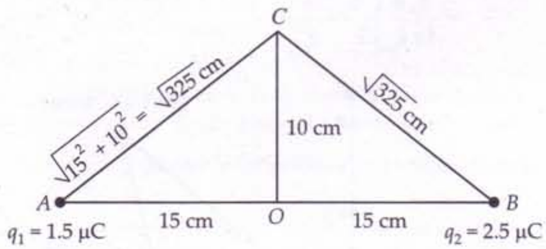


Fig. 2.191

(*a*) Here $r_1 = r_2 = 15\text{ cm} = 0.15\text{ m}$

∴ Potential at the midpoint O of the line joining the two charges is

$$V_0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1}+\frac{q_2}{r_2}\right]$$

$$= 9\times10^9\left[\frac{1.5\times10^{-6}}{0.15}+\frac{2.5\times10^{-6}}{0.15}\right]\text{V}$$

$$= 9\times10^9\times10^{-6}\left[10+\frac{50}{3}\right]\text{V}$$

$$= 9\times10^3\times\frac{80}{3}\text{ V} = \mathbf{2.4\times10^5\ V.}$$

(*b*) Here $r_1 = r_2 = \sqrt{10^2+15^2}$

$= \sqrt{325} \simeq 18\text{ cm} = 0.18\text{ m}$

∴ Potential at point C due to the two charges is

$$V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1}+\frac{q_2}{r_2}\right]$$

$$= 9\times10^9\left[\frac{1.5\times10^{-6}}{0.18}+\frac{2.5\times10^{-6}}{0.18}\right]$$

$$= \frac{9\times10^9\times4\times10^{-6}}{0.18} = \mathbf{2\times10^5\ V.}$$

2.15. *A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q.*

(*a*) *A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell ?*

(*b*) *Is the electric field inside a cavity (with no charge) zero even if the shell is not spherical, but has any irregular shape ? Explain.*

Ans. (*a*) The charge q placed at the centre of the shell induces a charge $-q$ on the inner surface of the shell and charge $+q$ on its outer surface.

∴ Surface charge density on the inner surface of the shell

$$= \frac{\text{charge}}{\text{surface area}} = -\frac{q}{4\pi r_1^2}$$

Surface charge density on the outer surface of the shell

$$= \frac{Q+q}{4\pi r_2^2}.$$

(*b*) Even if the shell is not spherical, the entire charge resides on its outer surface. The net charge on the inner surface enclosing the cavity is zero. From Gauss's theorem, electric field vanishes at all points inside the cavity. For a cavity of arbitrary shape, this is not enough to claim that electric field inside must be zero. The cavity surface may have positive and negative charges with total charge zero.

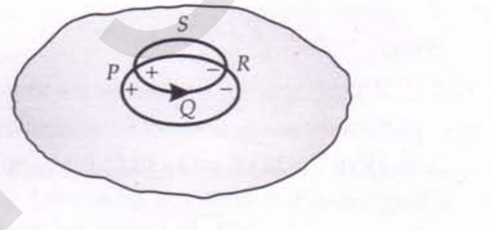


Fig. 2.192 Electric field vanishes inside a cavity of any shape.

To overrule this possibility, consider a closed loop *PQRSP*, such that part *PQR* is inside the cavity along a line of force and the part *RSP* is inside the conductor. Since the field inside a conductor is zero, this gives a network done by the field (in part *RSP*) in carrying a test charge over a closed loop. But this is not possible for a conservative field like the electrostatic field. Hence there are no lines of force (*i.e.*, no field), and no charge on the inner surface of the conductor, whatever be its shape.

2.16. *(a) Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by*

$$(\vec{E}_2-\vec{E}_1)\cdot\hat{n} = \frac{\sigma}{\varepsilon_0}$$

where $\hat{n}$ is a unit vector normal to the surface at a point and σ is the surface charge density at that point. (The direction of $\hat{n}$ is from side 1 to side 2)

Hence show that just outside a conductor, the electric field is $\sigma\hat{n}/\varepsilon_0$.

(b) Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another.

[**Hint :** For (*a*), Use Gauss's law. For (*b*), use the fact that work done by electrostatic field on a closed loop is zero.]

Ans. (*a*) Electric field near a plane sheet of charge is given by

$$E = \frac{\sigma}{2\varepsilon_0}$$

If $\hat{n}$ is a unit vector normal to the sheet from side 1 to side 2, then electric field on side 2

$$\vec{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

in the direction of the outward normal to the side 2.

Similarly, electric field on side 1 is

$$\vec{E}_1 = -\frac{\sigma}{2\varepsilon_0}\hat{n}$$

in the direction of the outward normal to the side 1.

$$\therefore \quad (\vec{E}_2 - \vec{E}_1).\hat{n} = \frac{\sigma}{2\varepsilon_0} - \left(-\frac{\sigma}{2\varepsilon_0}\right) = \frac{\sigma}{\varepsilon_0}$$

As $\vec{E}_1$ and $\vec{E}_2$ act in opposite directions, there must be discontinuity at the sheet of charge. Now electric field vanishes inside a conductor, therefore

$$\vec{E}_1 = 0$$

Hence outside the conductor, the electric field is

$$\vec{E} = \vec{E}_2 = \frac{\sigma}{\varepsilon_0}\hat{n}$$

(*b*) Let *XY* be the charged surface of a dielectric and $\vec{E}_1$ and $\vec{E}_2$ be the electric fields on the two sides of the charged surface as shown in Fig. 2.193.

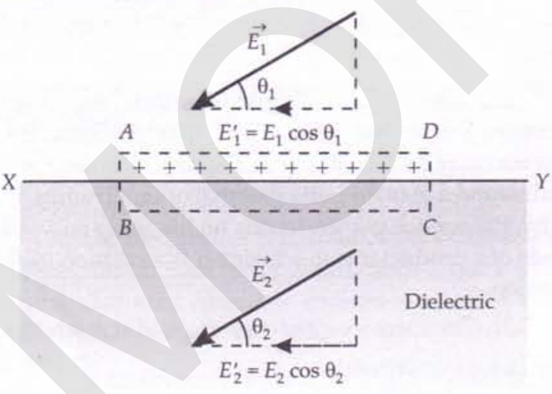


Fig. 2.193

Consider a rectangular loop *ABCD* with length *l* and negligibly small breadth. Line integral along the closed path *ABCD* will be

$$\int \vec{E}.\vec{dl} = \vec{E}_1.\vec{l} - \vec{E}_2.\vec{l} = 0$$

or $\quad E_1 l\cos\theta_1 - E_2 l\cos\theta_2 = 0$

$$(E_1\cos\theta_1 - E_2\cos\theta_2)\,l = 0$$

$$(E_1' - E_2')\,l = 0$$

where E_1' and E_2' are the tangential components of $\vec{E}_1$ and $\vec{E}_2$, respectively. Thus,

$$E_1' = E_2' \qquad (\because\ l \neq 0)$$

Hence the tangential component of the electrostatic field is continuous across the surface.

2.17. *A long charged cylinder of linear charged density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders ?*

Ans. Refer answer to Q. 35 on page 2.30.

2.18. *In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 Å.*

(*i*) *Estimate the potential energy of the system in eV, taking the zero of potential energy at infinite separation of the electron from proton.*

(*ii*) *What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (i) ?*

(*iii*) *What are the answers to (i) and (ii) above if the zero of potential energy is taken at 1.06 Å separation ?*

Ans. (*i*) $q_1 = -1.6\times10^{-19}$ C,

$q_2 = +1.6\times10^{-19}$ C,

$r = 0.53$ Å $= 0.53\times10^{-10}$ m

P.E. of the electron-proton system will be

$$U = \frac{1}{4\pi\varepsilon_0}.\frac{q_1 q_2}{r}$$

$$= 9\times10^9 \times \frac{(-1.6\times10^{-19})\times1.6\times10^{-19}}{0.53\times10^{-10}}\ \text{J}$$

$$= -\frac{9\times1.6\times1.6\times10^{-19}}{0.53}\ \text{J}$$

$$= -\frac{9\times1.6\times1.6\times10^{-19}}{0.53\times1.6\times10^{-19}}\ \text{eV}$$

$$\simeq \mathbf{-27.2\ eV.}$$

(*ii*) K.E. of the electron in the orbit

$$= \frac{1}{2}\text{P.E.} = \frac{1}{2}\times27.2\ \text{eV} = 13.6\ \text{eV}$$

$\therefore$ Total energy of the electron

$= \text{P.E.} + \text{K.E.}$

$= (-27.2 + 13.6)$ eV

$= \mathbf{-13.6\ eV.}$

As minimum energy of the free electron is zero, so minimum work required to free the electron

$= 0 - (-13.6)$

$= \mathbf{13.6\ eV.}$

(*iii*) When the zero of potential energy is not taken at infinity, the potential energy of the system is

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= 9 \times 10^9 \times (-1.6 \times 10^{-19}) \times 1.6 \times 10^{-19} \times \left[\frac{1}{0.53 \times 10^{-10}} - \frac{1}{1.06 \times 10^{-10}}\right] \text{J}$$

$$= -\frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{1.6 \times 10^{-19} \times 0.53 \times 10^{-10}}\left[1 - \frac{1}{2}\right] \text{eV}$$

$$= -\frac{9 \times 1.6}{0.53 \times 2} \text{eV} = -13.6 \text{ eV}$$

This indicates that the K.E. of 13.6 eV of case (i) is used up in increasing the P.E. from –27.2 eV to – 13.6 eV as the electron is carried from 0.53 Å to 1.06 Å position. K.E. in this situation should be zero. As the total energy in this case is zero, therefore, minimum work required to free the electron

$$= 0 - (-13.6 \text{ eV}) = \mathbf{13.6 \text{ eV}}.$$

2.19. *If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion (H_2^+). In the ground state of a H_2^+ ion, the two protons are separated by roughly 1.5 Å, and the electron is roughly 1 Å from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.*

Ans. The system of charges is shown in Fig. 2.194.

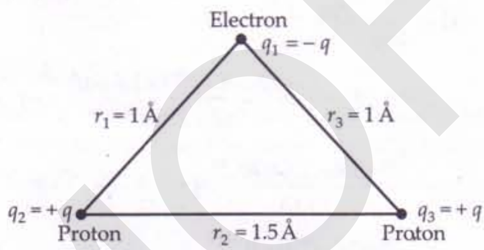


Fig. 2.194

Charge on an electron,

$$q_1 = -e = -1.6 \times 10^{-19} \text{ C}$$

Charge on each proton,

$$q_2 = q_3 = +e = +1.6 \times 10^{-19} \text{ C}$$

If the zero of potential energy is taken at infinity, then potential energy of the system is

$$U = U_{12} + U_{23} + U_{13} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_1} + \frac{q_2 q_3}{r_2} + \frac{q_1 q_3}{r_3}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{(-q)q}{1 \times 10^{-10}} + \frac{q.q}{1.5 \times 10^{-10}} + \frac{(-q)q}{1 \times 10^{-10}}\right]$$

$$= \frac{e^2}{4\pi\varepsilon_0 \times 10^{-10}}\left[-1 + \frac{1}{1.5} - 1\right] \quad [q = e]$$

$$= \frac{(1.6 \times 10^{-19})^2 \times 9 \times 10^9}{10^{-10}} \times \left(\frac{-4}{3}\right) \text{J}$$

$$= -\frac{(1.6 \times 10^{-19})^2 \times 9 \times 10^9 \times 4}{1.6 \times 10^{-19} \times 3} \text{eV} = \mathbf{-19.2 \text{ eV}}$$

$$[\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}]$$

2.20. *Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres ? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than that on its flatter portions.*

Ans. The charges will flow between the two spheres till their potentials become equal. Then the charges on the two spheres would be

$$\frac{Q_1}{Q_2} = \frac{C_1 V}{C_2 V} = \frac{C_1}{C_2}$$

But $\quad \frac{C_1}{C_2} = \frac{a}{b}$

$\therefore \quad \frac{Q_1}{Q_2} = \frac{a}{b}$

The ratio of the electric fields at the surface of the two spheres will be

$$\frac{E_1}{E_2} = \frac{\frac{1}{4\pi\varepsilon_0}.\frac{Q_1}{a^2}}{\frac{1}{4\pi\varepsilon_0}.\frac{Q_2}{b^2}} = \frac{Q_1}{Q_2}.\frac{b^2}{a^2} = \frac{a}{b}.\frac{b^2}{a^2} = \frac{b}{a}.$$

Also, $\quad \frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2}$

$\therefore \quad \frac{\sigma_1}{\sigma_2} = \frac{b}{a}$

Thus the surface charge densities are inversely proportional to the radii of the spheres. Since the flat portion may be considered as a spherical surface of large radius and a pointed portion as that of small radius, that is why, the surface charge density on the sharp and pointed ends of a conductor is much higher than that on its flatter portion.

2.21. *Two charges $-q$ and $+q$ are located at points $(0,0,-a)$ and $(0,0,a)$ respectively.*

(*i*) *What is the electrostatic potential at the points $(0,0,z)$ and $(x,y,0)$?*

(*ii*) *Obtain the dependence of potential on the distance r of a point from the origin when $r/a >> 1$.*

(*iii*) *How much work is done in moving a small test charge from the point (5, 0, 0) to $(-7,0,0)$ along the x-axis ?*

Does the answer change if the path of the test charge between the same points is not along the x-axis ?

Ans. (*i*) When the point *P* lies closer to the charge + *q* as shown in Fig. 2.195(*a*), the potential at this point *P* will be

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_1} - \frac{q}{r_2}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{z-a} - \frac{q}{z-(-a)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{2a}{z^2-a^2}$$

or $$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{z^2-a^2} \qquad [\because p = q\times 2a]$$

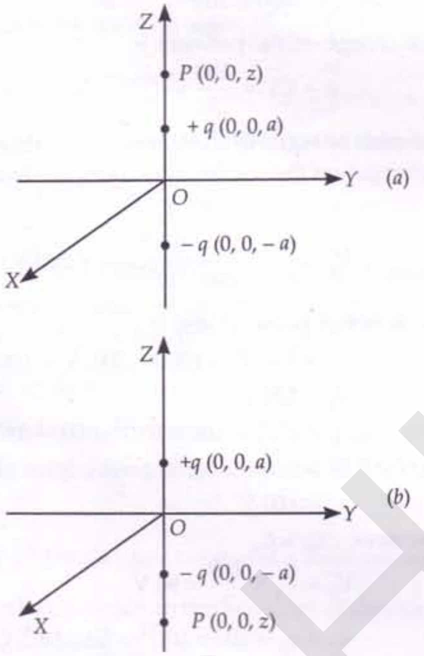


Fig. 2.195

When the point *P* lies closer to charge − *q*, as shown in Fig. 2.195(*b*), it can be easily seen that

$$V = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{z^2-a^2}$$

Again, any point $(x, y, 0)$ lies in *XY*-plane which is perpendicular bisector of *Z*-axis. Such a point will be at equal distances from the charges $-q$ and $+q$. Hence potential at point $(x, y, 0)$ will be **zero**.

(*ii*) If the distance of point *P* from the origin *O* is *r*, then from the results of part (*i*), we get

$$V = \pm\frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2-a^2} \qquad [\text{Put } z = r]$$

If $r >> a$, we can neglect a^2 compared to r^2, so

$$V = \pm\frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2}$$

$\therefore$ For $r >> a$, the dependence of potential *V* on *r* is $1/r^2$ type.

(*iii*) (5, 0, 0) and (− 7, 0, 0) are the points on the *X*-axis *i.e.*, these points lie on the perpendicular bisector of the dipole. Each point is at the same distance from the two charges. Hence electric potential at each of these points is zero.

Work done in moving the test charge q_0 from the point (5, 0, 0) to (− 7, 0, 0) is

$$W = q(V_1 - V_2) = q(0-0) = 0.$$

No, the answer will not change if the path of the test charge between the same two points is not along *X*-axis. This is because the work done by the electrostatic field between two points is independent of the path connecting the two points.

2.22. *Figure 2.196 below shows a charge array known as an electric quadrupole. For a point on the axis of the quadrupole, obtain the dependence of potential on r for r >> a. Contrast your result with that due to an electric dipole and an electric monopole (i.e. a single charge).*

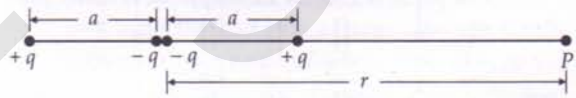


Fig. 2.196

Ans. Potential at point *P* is

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r-a} - \frac{q}{r} - \frac{q}{r} + \frac{q}{r+a}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot q\left[\frac{r(r+a) - 2(r-a)(r+a) + r(r-a)}{r(r-a)(r+a)}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot q\left[\frac{r^2 + ar - 2r^2 + 2a^2 + r^2 - ar}{r(r^2-a^2)}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2q\,a^2}{r(r^2-a^2)} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r(r^2-a^2)}$$

where $Q = 2q\,a^2$ is the *quadrupole moment* of the given charge distribution. As $r >> a$, so we can write

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^3}$$

Hence for large *r*, quadrupole potential varies as $1/r^3$, whereas dipole potential varies as $1/r^2$ and monopole potential varies as $1/r$.

2.23. *An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires a minimum number of capacitors.*

Ans. Let this arrangement require *n* capacitors of 1 μF each in series and *m* such series combinations to be connected in parallel.

P.D. across each capacitor of a series combination

$$= \frac{1000}{n} = 400 \quad \text{or} \quad n = \frac{1000}{400} = 2.5$$

But number of capacitors cannot be a fraction,

$\therefore \quad n = 3$

Equivalent capacitance of the combination is

$$\frac{1}{n} . m = 2 \quad \text{or} \quad m = 2n = 6$$

$\therefore$ Total number of capacitors required

$$= 3 \times 6 = 18$$

So six series combinations, each of three 1 μF capacitors, should be connected in parallel as shown in Fig. 2.197.

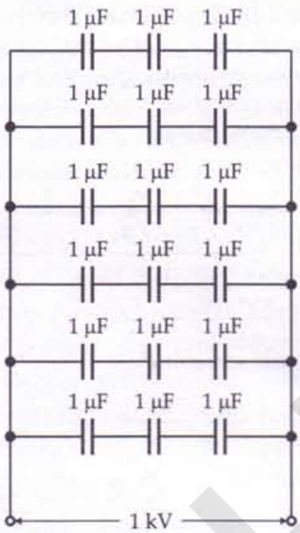


Fig. 2.197

2.24. *What is the area of the plates of a 2 F parallel plate capacitor ? Given that the separation between the plates is 0.5 cm.*

Ans. Here $C = 2\text{F}$, $d = 0.5\text{ cm} = 5 \times 10^{-3}\text{ m}$

As $\quad C = \frac{\varepsilon_0 A}{d}$

$$\therefore \quad A = \frac{Cd}{\varepsilon_0} = \frac{2 \times 5 \times 10^{-3}}{8.85 \times 10^{-12}} \text{ m}^2$$

$$\simeq 1130 \times 10^6 \text{ m}^2 = \mathbf{1130 \text{ km}^2}.$$

2.25. *Obtain the equivalent capacitance of the network shown in Fig. 2.198. For a 300 V supply, determine the charge and voltage across each capacitor.* **[CBSE OD 08]**

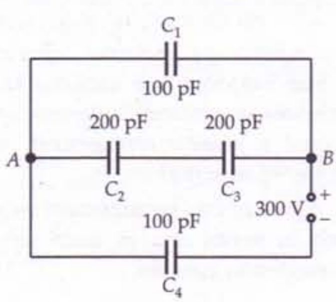


Fig. 2.198

Ans. As C_2 and C_3 are in series, their equivalent capacitance

$$= \frac{C_2 C_3}{C_2 + C_3} = \frac{200 \times 200}{200 + 200} = 100 \text{ pF}$$

Series combination of C_2 and C_3 is in parallel with C_1, their equivalent capacitance

$$= 100 \text{ pF} + 100 \text{ pF} = 200 \text{ pF}$$

The combination of C_1, C_2 and C_3 is in series with C_4, equivalent capacitance of the network

$$= \frac{200 \times 100}{200 + 100} \text{ pF} = \frac{200}{3} \text{ pF}$$

Total charge on the network is

$$q = CV = \frac{200}{3} \times 10^{-12} \times 300 = 2 \times 10^{-8} \text{ C}$$

This must be equal to charge on C_4 and also to the sum of the charges on the combination of C_1, C_2 and C_3.

$$\therefore \quad q_4 = q = 2 \times 10^{-8} \text{ C}$$

$$V_4 = \frac{q_4}{C_4} = \frac{2 \times 10^{-8}}{100 \times 10^{-12}} \text{ V} = \mathbf{200 \text{ V}}$$

P.D. between points A and B

$$= V - V_4 = (300 - 200) \text{ V} = 100 \text{ V}$$

$$\therefore \quad V_1 = \mathbf{100 \text{ V}}$$

$$q_1 = C_1 V_1 = 100 \times 10^{-12} \times 100 = \mathbf{10^{-8} \text{ C}}$$

Also the P.D. across the series combination of C_2 and C_3

$$= 100 \text{ V}$$

Now since $\quad C_2 = C_3$

$$\therefore \quad V_2 = V_3 = \frac{100}{2} = \mathbf{50 \text{ V}}$$

and $\quad q_2 = q_3 = 200 \times 10^{-12} \times 50 = \mathbf{10^{-8} \text{ C}}.$

2.26. *The plates of a parallel plate capacitor have an area of 90 cm² each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.*

(*i*) *How much energy is stored by the capacitor ?*

(*ii*) *View this energy stored in the electrostatic field between the plates and obtain the energy per unit volume u. Hence arrive at a relation between u and the magnitude of electric field E between the plates.*

Ans. (*i*) Here $A = 90 \text{ cm}^2 = 90 \times 10^{-4} \text{ m}^2 = 9 \times 10^{-3} \text{m}^2$

$$d = 2.5 \text{ mm} = 2.5 \times 10^{-3} \text{m},$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \text{Fm}^{-1},$$

$$V = 400 \text{ V}$$

Capacitance of the parallel plate capacitor is

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 9 \times 10^{-3}}{2.5 \times 10^{-3}} \text{ F}$$

$$= 31.86 \times 10^{-12} \text{ F} = \mathbf{31.86 \text{ pF}}.$$

Electrostatic energy stored by the capacitor,

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times 31.86 \times 10^{-12} \times (400)^2 \text{ J}$$

$$= 254.88 \times 10^{-8} \text{ J} = \mathbf{2.55 \times 10^{-6} \text{ J}}.$$

(*ii*) Energy stored per unit volume or energy density of the capacitor is

$$u = \frac{U}{Ad} = \frac{2.55 \times 10^{-6}}{9 \times 10^{-3} \times 2.5 \times 10^{-3}}\ \text{Jm}^{-3}$$

$$= \mathbf{0.113\ Jm^{-3}}.$$

The relation between u and E can be arrived at as follows :

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{1}{2}\frac{\varepsilon_0 A}{d}\cdot\frac{V^2}{Ad} = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2$$

or $$u = \frac{1}{2}\varepsilon_0 E^2.$$

2.27. *A 4 μF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 2 μF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation ?* **[CBSE OD 05]**

Ans. Initial electrostatic energy of the 4 μF capacitor is

$$U_i = \frac{1}{2}CV^2 = \frac{1}{2} \times 4 \times 10^{-6} \times (200)^2 = 8 \times 10^{-2}\ \text{J}$$

Charge on 4 μF capacitor

$$= CV = 4 \times 10^{-6} \times 200 = 8 \times 10^{-4}\ \text{C}$$

When the 4 μF and 2μF capacitors are connected together, both attain a common potential V. Thus

$$V = \frac{\text{Total charge}}{\text{Total capacitance}} = \frac{8 \times 10^{-4}\ \text{C}}{(4+2) \times 10^{-6}\ \text{F}} = \frac{400}{3}\ \text{V}$$

Final electrostatic energy of the combination,

$$U_f = \frac{1}{2} \times (4+2) \times 10^{-6} \times \left(\frac{400}{3}\right)^2\ \text{J} = \frac{16}{3} \times 10^{-2}\ \text{J}$$

$$= 5.33 \times 10^{-2}\ \text{J}$$

Electrostatic energy of the first capacitor lost in the form of heat and electromagnetic radiation is

$$\Delta U = U_i - U_f = (8 - 5.33) \times 10^{-2}\ \text{J}$$

$$= \mathbf{2.67 \times 10^{-2}\ J}.$$

2.28. *Show that the force on each plate of a parallel plate capacitor has a magnitude equal to* $\frac{1}{2}qE$*, where q is the charge on the capacitor, and E is the magnitude of electric field between the plates. Explain the origin of the factor* $\frac{1}{2}$.

Ans. Let A be the plate area and σ, the surface charge density of the capacitor. Then

$$q = \sigma A$$

$$E = \frac{\sigma}{\varepsilon_0}$$

Suppose we increase the separation of the capacitor plates by small distance Δx against the force F. Then work done by the external agency $= F.\Delta x$

If u be the energy stored per unit volume or the energy density of the capacitor, then increase in potential energy of the capacitor

$$= u \times \text{increase in volume} = u.A.\Delta x$$

$$\therefore \quad F.\Delta x = u.A.\Delta x$$

or $$F = uA = \frac{1}{2}\varepsilon_0 E^2 . A = \frac{1}{2}(\varepsilon_0 E)AE$$

$$= \frac{1}{2}.\sigma A.E = \frac{1}{2}qE$$

The physical origin of the factor $\frac{1}{2}$ in the force formula lies in the fact that just inside the capacitor, field is E, and outside it is zero. So the average value $E/2$ contributes to the force.

2.29. *A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports (Fig. 2.199). Show that the capacitance of a spherical capacitor is given by*

$$C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$$

where r_1 and r_2 are the radii of outer and inner spheres, respectively.

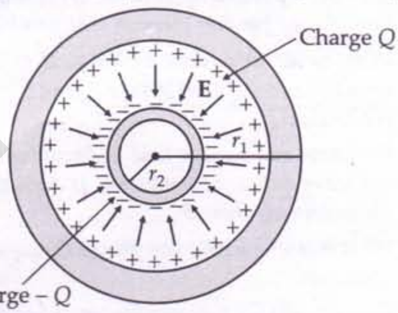


Fig. 2.199

Ans. Refer answer to Q. 34 on page 2.30.

2.30. *A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 μC. The space between the co-centric spheres is filled with a liquid of dielectric constant 32. (a) Determine the capacitance of the capacitor. (b) What is the potential of the inner sphere ? (c) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.*

Ans. Here $a = 12\ \text{cm} = 12 \times 10^{-2}\ \text{m}$,

$b = 13\ \text{cm} = 13 \times 10^{-2}\ \text{cm}$,

$q = 2.5\ \mu\text{C} = 2.5 \times 10^{-6}\ \text{C}$, $\kappa = 32$

(*a*) Capacitance of the spherical capacitor is

$$C = 4\pi\varepsilon_0\ \kappa.\frac{ab}{b-a}$$

$$= \frac{32}{9 \times 10^9}\cdot\frac{12 \times 10^{-2} \times 13 \times 10^{-2}}{(13-12) \times 10^{-2}}\ \text{F}$$

$$= \frac{32 \times 12 \times 13}{9} \times 10^{-11}\ \text{F} = \mathbf{5.5 \times 10^{-9}\ F.}$$

(*b*) Potential of the inner sphere is

$$V = \frac{q}{C} = \frac{2.5 \times 10^{-6}}{5.5 \times 10^{-9}} \text{ V} = 0.45 \times 10^{3} \text{ V} = \mathbf{4.5 \times 10^{2} \text{ V}}.$$

(*c*) Capacitance of the isolated sphere of radius 12 cm is

$$C = 4\pi\varepsilon_0 R = \frac{12 \times 10^{-2}}{9 \times 10^{9}} \text{ F} = \mathbf{1.3 \times 10^{-11} \text{ F}}.$$

When an earthed conductor is placed near a charged conductor, the capacitance of the latter increases. The two conductors form a capacitor. But the capacitance of an isolated conductor is always small.

2.31. *Answer carefully :*

(*i*) *Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by $\frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$, where r is the distance between their centres ?*

(*ii*) *If Coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss' law be still true ?*

(*iii*) *A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the line of force passing through that point ?*

(*iv*) *What is the work done by the field of a nucleus in a complete circular orbit of the electron ? What if the orbit is elliptical ?*

(*v*) *We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there ?*

(*vi*) *What meaning would you give to the capacity of a single conductor ?*

(*vii*) *Guess a possible reason why water has a much greater dielectric constant (= 80) than say, mica (= 6).*

Ans. (*i*) No. When the two spheres are brought close to each other, their charge distributions do not remain uniform and they will not act as point charges.

(*ii*) No. Gauss's law will not hold if Coulomb's law involved $1/r^3$ or any other power of *r* (except 2). In that case the electric flux will depend upon *r* also.

(*iii*) Not necessarily. The small test charge will move along the line of force only if it is a straight line. The *line of force gives the direction of acceleration, and not that of velocity.*

(*iv*) Zero. But when the orbit is elliptical, work is done in moving the electron from one point to the other. However, net work done over a complete cycle is zero.

(*v*) No, potential is everywhere constant as it is a scalar quantity.

(*vi*) A single conductor is a capacitor with one plate at infinity. It also possesses capacitance.

(*vii*) Because of its bent shape and the presence of two highly polar O – H bonds, a water molecule possesses a large permanent dipole moment about 0.6×10^{-29} Cm. Hence water has a large dielectric constant.

2.32. *A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e., bending of field lines at the ends).*

Ans. Here $L = 15 \text{ cm} = 0.15 \text{ m}$,
$q = 3.5\ \mu\text{C} = 3.5 \times 10^{-6}$ C, $a = 1.4 \text{ cm} = 0.014$ m,
$b = 1.5 \text{ cm} = 0.015$ m

Capacitance of a cylindrical capacitor is given by

$$C = \frac{2\pi\varepsilon_0 L}{\ln\frac{b}{a}} = \frac{L}{2\frac{1}{4\pi\varepsilon_0} 2.303 \log\frac{b}{a}}$$

$$= \frac{0.15}{2 \times 9 \times 10^{9} \times 2.303 \log\frac{0.015}{0.014}} \text{ F}$$

$$= \frac{0.15 \times 10^{-9}}{18 \times 2.303 \times 0.03} \text{ F}$$

$$= 0.1206 \times 10^{-9} \text{ F} = \mathbf{1.2 \times 10^{-10} \text{ F}}$$

Potential,

$$V = \frac{q}{C} = \frac{3.5 \times 10^{-6}}{1.2 \times 10^{-10}} \text{ V} = \mathbf{2.9 \times 10^{4} \text{ V}}.$$

2.33. *A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about 10^7 Vm^{-1}. For safety, we would like the field never to exceed say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 pF ?* [CBSE OD 05]

Ans. Maximum permissible voltage

$$= 1 \text{ kV} = 10^{3} \text{ V}$$

Maximum permissible electric field

$$= 10\% \text{ of } 10^{7} \text{ Vm}^{-1} = 10^{6} \text{ Vm}^{-1}$$

∴ Minimum separation *d* required between the plates is given by

$$E = \frac{V}{d} \quad \text{or} \quad d = \frac{V}{E} = \frac{10^{3}}{10^{6}} = 10^{-3} \text{ m}$$

Capacitance of a parallel plate capacitor is

$$C = \frac{\kappa\varepsilon_0 A}{d}$$

$$\therefore \quad A = \frac{Cd}{\kappa\varepsilon_0} = \frac{50 \times 10^{-12} \times 10^{-3}}{3 \times 8.85 \times 10^{-12}} \text{ m}^2$$

$$= 18.8 \times 10^{-4} \text{ m}^2 \simeq \mathbf{19 \text{ cm}^2}.$$

2.34. *Describe schematically the equipotential surfaces corresponding to*

(*i*) *a constant electric field in the Z-direction.*

(*ii*) *a field that uniformly increases in magnitude but remains in a constant (say, Z) directions.*

(*iii*) *a single positive charge at the origin.*

(*iv*) *a uniform grid consisting of long equally spaced parallel charged wires in a plane.*

Ans. (*i*) For a constant electric field in Z-direction, equipotential surfaces will be planes parallel to XY-planes, as shown in Fig. 2.200.

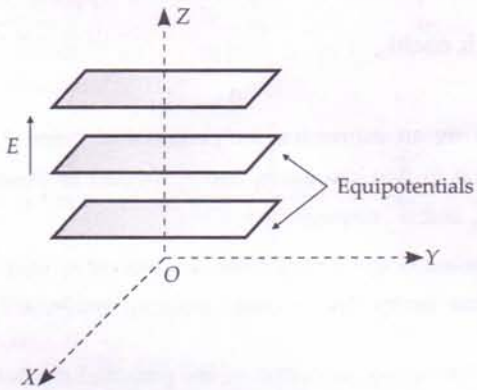


Fig. 2.200

(*ii*) In this case also, the equipotential surfaces will be planes parallel to XY-plane. However, as field increases, such planes will get closer.

(*iii*) For a single positive charge at the origin, the equipotential surfaces will be concentric spheres having origin as their common centre, as shown in Fig. 2.25. The separation between the equipotentials differing by a constant potential increases with increase in distance from the origin.

(*iv*) Near the grid the equipotential surfaces will have varying shapes. At far off distances, the equipotential surfaces will be planes parallel to the grid.

2.35. *In a Van de Graaff type generator, a spherical metal shell is to be a 1.5×10^6 V electrode. The dielectric strength of the gas surrounding the electrode is 5×10^7 Vm^{-1}. What is the minimum radius of the spherical shell required ?* **[CBSE OD 08]**

Ans. Maximum permissible potential, $V = 1.5 \times 10^6$ V

For safety, the maximum permissible electric field is

E = 10% of dielectric strength

= 10% of 5×10^7 Vm^{-1} = 5×10^6 Vm^{-1}

Now for a spherical shell,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} = \frac{V}{r}$$

$\therefore$ Minimum radius required is

$$r = \frac{V}{E} = \frac{1.5 \times 10^6 \text{ V}}{5 \times 10^6 \text{ Vm}^{-1}} = 3 \times 10^{-1} \text{ m} = \mathbf{30\ cm.}$$

2.36. *A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.*

Ans. Refer answer to Q. 55 on page 2.67.

2.37. *Answer the following :*

(*i*) *The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 Vm^{-1}. Why do then we not get an electric shock as we step out of our house into the open ? (Assume the house to be a steel cage so there is no field inside.)*

(*ii*) *A man fixes outside his house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area 1 m^2. Will he get an electric shock if he touches the metal sheet next morning ?*

(*iii*) *The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral ? In other words, what keeps the atmosphere charged ?*

(*iv*) *What are the forms of energy into which the electric energy of the atmosphere is dissipated during a lightning ?*

Ans. (*i*) Normally the equipotential surfaces are parallel to the surface of the earth as shown in Fig. 2.201. Now our body is a good conductor. So as we step out into the open, the original equipotential surfaces of open air get modified, but keeping our head and the ground at the same potential and we do not get any electric shock.

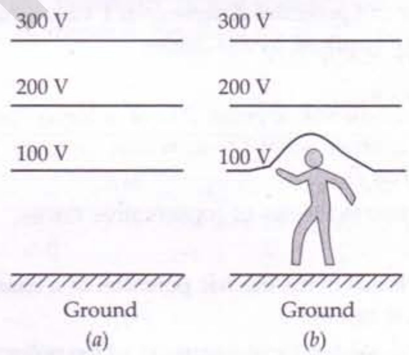


Fig. 2.201

(*ii*) Yes. The aluminium sheet and the ground form a capacitor with insulating slab as dielectric. The discharging current in the atmosphere will charge the capacitor steadily and raise its voltage. Next morning, if the man touches the metal sheet, he will receive shock to the extent depending upon the capacitance of the capacitor formed.

(*iii*) The atmosphere is continuously being charged by thunder storms and lightning bolts all over globe and maintains an equilibrium with the discharge of the atmosphere in ordinary weather conditions.

(*iv*) The electrical energy is lost as (*i*) light energy involved in lightning (*ii*) heat and sound energy in the accompanying thunder.

Text Based Exercises

TYPE A : VERY SHORT ANSWER QUESTIONS (1 mark each)

1. Define electric potential. Is it a scalar or a vector quantity ? [Punjab 01 ; CBSE OD 06]
2. Define the unit of electric potential. [Punjab 02]
3. Write down the relation between electric field and electric potential at a point.
4. Name the physical quantity whose SI, unit is JC^{-1}. Is it a scalar or a vector quantity ? [CBSE OD 2010]
5. Write the SI unit of potential gradient.
6. Define electric potential difference between two points. Is it scalar or vector ? [Punjab 01]
7. What do you mean by a potential difference of 1 volt ?
8. Write the dimensional formula of potential difference.
9. 5 J of work is done in moving a positive charge of 0.5 C between two points. What is the potential difference between these two points ? [ISCE 95]
10. A charge of 2 C moves between two points maintained at a potential difference of 1 volt. What is the energy acquired by the charge ? [ISCE 94 ; CBSE D 10C]
11. In a conductor, a point P is at a higher potential than another point Q. In which direction do the electrons move ?
12. Give two examples of conservative forces. [Himachal 93]
13. How much is the electric potential of a charge at a point at infinity ?
14. What is the nature of symmetry of the potential of a point charge ?
15. What are the points at which electric potential of a dipole has maximum value ?
16. What are the points at which electric potential of a dipole has a minimum value ?
17. What is the nature of symmetry of a dipole potential ?
18. What is electrostatic potential energy ? Where does it reside ?
19. What is the value of the angle between the vectors $\vec{p}$ and $\vec{E}$ for which the potential energy of an electric dipole of dipole moment $\vec{p}$, kept in an external electric field $\vec{E}$, has maximum value ? [CBSE SP 15]
20. Write an expression for potential at point $P(\vec{r})$ due to two charges q_1 and q_2 located at positions $\vec{r}_1$ and $\vec{r}_2$ respectively.
21. Define electron volt. How is it related to joule ?
22. How many electron volts make up one joule ? [Himachal 93]
23. Will there be any effect on the potential at a point if the medium around this point is changed ?
24. What work must be done in carrying an α-particle across a potential difference of 1 volt ?
25. What is an equipotential surface ? Give an example. [Punjab 2000, 02 ; CBSE D 03]
26. Why are electric field lines perpendicular at a point on an equipotential surface of a conductor ? [CBSE OD 15C]
27. Can you say that the earth is an equipotential surface ?
28. What is the geometrical shape of equipotential surfaces due to a single isolated charge ? [CBSE D13]
29. What is the shape of the equipotential surfaces for a uniform electric field ? [Punjab 01]
30. How much work is done in moving a 500 μC charge between two points on an equipotential surface ? [CBSE D 02]
31. A charge of + 1C is placed at the centre of a spherical shell of radius 10 cm. What will be the work done in moving a charge of + 1 μC on its surface through a distance of 5 cm ?
32. What is the optical analogue of an equipotential surface ?
33. The middle point of a conductor is earthed and its ends are maintained at a potential difference of 220 V. What is the potential at the ends and at the middle point ?
34. Define capacitance of a conductor.
35. Can there be a potential difference between two conductors of same volume carrying equal positive charges ?
36. The capacitance of a conductor is 1 farad. What do you mean by this statement ?
37. What is a capacitor ? [Punjab 96C]

38. Write the physical quantity that has its unit coulomb volt^{-1}. Is it a vector or scalar quantity ? [CBSE D 93C, 98]
39. Define capacitance. Give its SI unit. [CBSE D 93C ; ISCE 98]
40. Define SI unit of capacitance. [CBSE OD 94]
41. Write the dimensions of capacitance.
42. What is the net charge on a charged capacitor ?
43. On what factors does the capacitance of a capacitor depend ?
44. Write two applications of capacitors in electrical circuits.
45. In what form is the energy stored in a charged capacitor ?
46. What is the basic purpose of using a capacitor ?
47. Write different expressions for the energy stored in a capacitor.
48. Write down the expression for the capacitance of a spherical capacitor.
49. The difference between the radii of the two spheres of a spherical capacitor is increased. Will the capacitance increase or decrease ? [Punjab 2000]
50. What is a dielectric ?
51. Define dielectric constant in terms of the capacitance of a capacitor. [CBSE D 06]
52. Write down the relation between dielectric constant and electric susceptibility.
53. Write a relation for polarisation $\vec{P}$ of a dielectric material in the presence of an external electric field $\vec{E}$. [CBSE OD 15]
54. Define dielectric strength of a medium. What is its value for vacuum ?
55. Where is the knowledge of dielectric strength helpful ?
56. What is the effect of temperature on dielectric constant ?
57. An air capacitor is given a charge of 2 μC raising its potential to 200 V. If on inserting a dielectric medium, its potential falls to 50 V, what is the dielectric constant of the medium ?
58. An uncharged insulated conductor A is brought near a charged insulated conductor B. What happens to the charge and potential of B? [CBSE OD 01C]
59. For a given potential difference, does a capacitor store more or less charge with a dielectric than it does without a dielectric ?
60. Can we place a parallel plate capacitor of 1 F capacity in our house ?
61. What is the basic difference between a capacitor and an electric cell ?
62. Two capacitors of capacitances C_1 and C_2 are connected in parallel. A charge q is given to the combination. What will be the potential difference across each capacitor ?
63. What is the order of capacitances used in a radio receiver ?
64. Is there any conductor which can take unlimited charge ?
65. A parallel plate capacitor with air between the plates has a capacitance of 8 pF. What will be the capacitance if the distance between the plates be reduced by half and the space between them is filled with a substance of dielectric constant $\kappa = 6$? [CBSE D 05]
66. A 500 μC charge is at the centre of a square of side 10 cm. Find the work done in moving a charge of 10 μC between two diagonally opposite points on the square. [CBSE D 08]
67. The graph of Fig. 2.202, shows the variation of the total energy (E) stored in a capacitor against the value of the capacitance (C) itself. Which of the two the charge on the capacitor or the potential used to charge it is kept constant for this graph ? [CBSE Sample Paper 08]

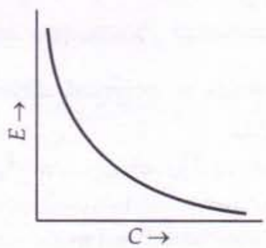


Fig. 2.202

68. Define the term 'potential energy' of charge 'q' at a distance 'r' in an external electric field. [CBSE OD 09]
69. What is the work done in moving a test charge q through a distance of 1 cm along the equatorial axis of an electric dipole ? [CBSE OD 09]
70. What is the electrostatic potential due to an electric dipole at an equatorial point ? [CBSE OD 09]
71. A metal plate is introduced between the plates of a charged parallel plate capacitor. What is its effect on the capacitance of the capacitor ? [CBSE F 09]

72. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. What is the potential at the centre of the sphere ? [CBSE OD 11]

73. In which orientation, a dipole placed in a uniform electric field is in (*i*) stable, (*ii*) unstable equilibrium ? [CBSE OD 08 ; D 10]

74. Write the expression for the work done on an electric dipole of dipole moment $\vec{p}$ in turning it from its position of stable equilibrium to a position of unstable equilibrium in a uniform electric field $\vec{E}$. [CBSE D13C]

75. Two charges 2 μC and –2 μC are placed at points *A* and *B*, 5 cm apart. Depict an equipotential surface of the system. [CBSE D13C]

76. What is the amount of work done in moving a charge around a circular arc of radius *r* at the centre of which another point charge is located ? [CBSE OD13C]

77. What is the equivalent capacitance, *C*, of the five capacitors, connected as shown in Fig. 2.203 ? [CBSE Sample Paper 2011]

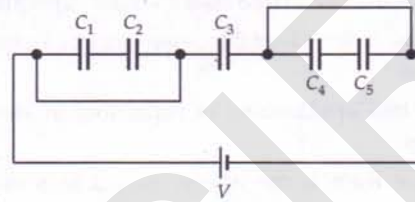


Fig. 2.203

Answers

1. The electric potential at any point in an electric field is defined as the amount of work done in moving a unit positive charge from infinity to that point against the electrostatic force. It is a scalar quantity.

2. The SI unit of electric potential is volt. The electric potential at a point is said to be 1 volt, if 1 joule of work is done in moving a positive charge of 1 coulomb from infinity to that point against the electrostatic force.

3. $E = -\frac{dV}{dr}$.

4. Electric potential or potential difference. It is a scalar quantity.

5. SI unit of potential gradient = Vm^{-1}.

6. The potential difference between two points in an electric field may be defined as the amount of work done in moving a unit positive charge from one point to the other against the electrostatic force. It is a scalar.

7. The potential difference between two points is said to be 1 volt if 1 joule of work is done in moving a positive charge of 1 coulomb from one point to the other against the electrostatic force.

8. Potential difference $= \frac{\text{work done}}{\text{charge}} = \frac{ML^2T^{-2}}{C}$

$$= \frac{ML^2T^{-2}}{AT} = [ML^2T^{-3}A^{-1}]$$

9. $V = \frac{W}{q} = \frac{5\,J}{0.5\,C} = \mathbf{10\ V}$.

10. Energy acquired by the charge $= qV = 2\,C \times 1\,V = \mathbf{2\ J}$.

11. From *Q* to *P*.

12. (*i*) Electrostatic force, (*ii*) Gravitational force.

13. Zero.

14. The potential of a point charge is spherically symmetric.

15. At axial points, the electric potential of a dipole has a maximum positive or negative value.

16. At equatorial points, the electric potential of a dipole is zero.

17. The dipole potential is cylindrically symmetric.

18. The electrostatic potential energy of a system of charges may be defined as the work required to be done to bring the various charges to their respective positions from infinity.

19. P.E. $= -pE\cos\theta$. Clearly, P.E. is maximum when $\cos\theta = -1$ or $\theta = \mathbf{180°}$.

20. $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{|\vec{r} - \vec{r}_1|} + \frac{q_2}{|\vec{r} - \vec{r}_2|}\right]$

21. Electron volt is the potential energy gained or lost by an electron in moving through a potential difference of one volt.

$$1\,\text{electron volt} = 1\,\text{eV} = 1.6 \times 10^{-19}\ \text{J}$$

22. $1\,J = 6.25 \times 10^{18}$ eV.

23. Yes. If the dielectric constant of the medium is increased, the electric potential will decrease.

24. $W = q\Delta V = 2e\,\Delta V$

$$= 3.2 \times 10^{-19}\ C \times 1\ V = 3.2 \times 10^{-19}\ J.$$

25. Any surface which has same electric potential at every point is called an equipotential surface. The surface of a charged conductor is an equipotential surface.

26. If it were not so, the presence of a component of the field along the surface would destroy its equipotential nature.

27. Yes. Earth is a conductor, so its surface is equipotential.

28. For a point charge, the equipotential surfaces are concentric spherical shells with their centre at the point charge.

29. For a uniform electric field, the equipotential surfaces are parallel planes perpendicular to the direction of the electric field.

30. Zero.

$$W = q\,\Delta V = 500\,\mu\text{C} \times 0 = \mathbf{0}.$$

31. Zero. This is because the surface of the spherical shell will be an equipotential surface.

32. Wavefront.

33. The potential at the middle point of the conductor is zero and that at the ends + 110 V and − 110 V, so that the p.d. at ends = 110 − (− 110) = 220 V.

34. The capacitance of a conductor may be defined as the charge required to raise its potential by unit amount.

35. Yes. Two conductors of same volume but of different shapes will have different capacitances.

36. A conductor is said to have a capacitance of 1 farad, if 1 coulomb of charge increases its electric potential through 1 volt.

37. A capacitor is a device to store electric charge. It consists of two conducting plates separated by an insulating medium.

38. Capacitance has its unit coulomb volt^{-1}. It is a scalar quantity.

39. The capacitance of a capacitor may be defined as the charge required to be supplied to either of the conductors so as to increase the potential difference between them by unit amount.

40. The SI unit of capacitance is farad (F). A capacitor has a capacitance of 1 F if 1 coulomb of charge is transferred from its one plate to another on applying a potential difference of 1 volt across the two plates.

41. As $1\,\text{F} = \dfrac{1\text{C}}{1\text{V}} = \dfrac{1\text{C}}{1\text{J}/\text{C}} = \dfrac{1\text{C}^2}{1\text{J}} = \dfrac{1(\text{As})^2}{1\text{Nm}}$

$$\therefore \quad [\text{Capacitance}] = \frac{\text{A}^2\text{T}^2}{\text{MLT}^{-2}\text{L}} = [\text{M}^{-1}\text{L}^{-2}\text{T}^4\text{A}^2].$$

42. Zero, because the two plates have equal and opposite charges.

43. The capacitance of a capacitor depends on the geometry of the plates, distance between them and the nature of the dielectric medium between them.

44. (*i*) Capacitors are used in radio circuits for tuning purposes.
(*ii*) Capacitors are used in power supplies for smoothening the rectified current.

45. In a charged capacitor, energy is stored in the form of electrostatic potential energy in the electric field between its plates.

46. To store charge and electric energy.

47. $U = \dfrac{1}{2}CV^2 = \dfrac{1}{2}\dfrac{Q^2}{V} = \dfrac{1}{2}QC.$

48. $C = 4\pi\varepsilon_0 \cdot \dfrac{ab}{b-a}$, where a and b are the radii of the inner and outer spheres respectively.

49. The capacitance will increase.

50. A dielectric is essentially an insulator which allows electric induction to take place through it but does not permit the flow of charges through it.

51. The ratio of the capacitance (C_d) of the capacitor completely filled with the dielectric material to the capacitance (C_v) of the same capacitor with vacuum between its plates is called dielectric constant.

$$\kappa = \frac{C_d}{C_v}$$

52. $\kappa = 1 + \chi$, where κ is dielectric constant and χ is electric susceptibility.

53. $\vec{P} = \varepsilon_0 \chi_e \vec{E}$.

54. The maximum value of electric field that can exist inside a dielectric without causing its electrical breakdown is called its dielectric strength. The dielectric strength for vacuum is infinity.

55. The knowledge of dielectric strength helps in designing a capacitor by determining the maximum potential that can be applied across the capacitor without causing its electrical breakdown.

56. The value of dielectric constant decreases with the increase of temperature.

57. $\kappa = \dfrac{V_{\text{vacuum}}}{V_{\text{dielectric}}} = \dfrac{200}{50} = \mathbf{4}.$

58. The charge on the conductor B remains unchanged but its potential gets lowered.

59. A capacitor with a dielectric has a higher capacitance and hence stores more charge.

60. No. If $d = 1\text{cm} = 10^{-2}$ m, then area of such a capacitor would be

$$A = \frac{Cd}{\varepsilon_0} = \frac{1\,\text{F} \times 10^{-2}\ \text{m}}{8.85 \times 10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}} = 10^9\ \text{m}^2$$

This is a plate about 30 km in length and breadth.

61. A capacitor provides electrical energy stored in it. A cell provides electrical energy by converting chemical energy into electrical energy.

62. In parallel combination, potential difference is same across each capacitor.

Net capacitance, $C = C_1 + C_2$

$\therefore$ P.D. across each capacitor, $V = \frac{q}{C} = \frac{q}{C_1 + C_2}$.

63. In the power supply, it is $1 - 10\,\mu\text{F}$ and for tuning purposes, it is $100\,\mu\text{F}$.

64. Yes, the earth because of its large capacitance can take unlimited charge.

65. With air between the capacitor plates,

$$C_0 = \frac{\varepsilon_0 A}{d} = 8\ \text{pF}$$

With dielectric between the capacitor plates,

$$C = \kappa \frac{\varepsilon_0 A}{d/2} = 2\kappa C_0 = 2 \times 6 \times 8 = \mathbf{96\ pF.}$$

66. The work done in moving a charge of $10\,\mu\text{C}$ between two diagonally opposite points on the square will be zero because these two points will be equipotential.

67. Energy stored, $E = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$

When Q is constant, $E \propto \frac{1}{C}$, and we get a graph of the type given in the question.

Hence the charge Q on the capacitor is kept constant.

68. The potential energy of a charge q is the work done in bringing charge q from infinity to the position $\vec{r}$ in the external electric field.

$$U(\vec{r}) = qV(\vec{r})$$

69. As potential at any point on the equatorial axis of an electric dipole is zero, so

$$W = q\Delta V = q(0-0) = 0.$$

70. Zero.

71. The introduction of a metal sheet of thickness t in a parallel plate capacitor increases its capacitance by a factor of $\frac{d}{d-t}$, where d is the plate separation of the capacitor.

72. Potential at the centre

= Potential at the surface = 10V.

73. (*i*) When the dipole moment $\vec{p}$ is parallel to the the electric field $\vec{E}(\theta = 0°)$, the dipole is in stable equilibrium.

(*ii*) When the dipole moment $\vec{p}$ is antiparallel to the electric field $\vec{E}(\theta = 180°)$, the dipole is in unstable equilibrium.

74. $W = pE(\cos\theta_1 - \cos\theta_2) = pE(\cos 0° - \cos 180°)$
$= pE(1+1) = \mathbf{2pE}$.

75. See Fig. 2.26 on page 2.15.

76. Zero, because all points of the circular arc will be at the same potential.

77. $C = C_3$, because the combinations of C_1 and C_2 as well as C_4 and C_5 have been shorted.

TYPE B : SHORT ANSWER QUESTIONS (2 or 3 marks each)

1. Distinguish between electric potential and potential energy and write the relation between them. [Punjab 96C]

2. Define electric potential. Derive an expression for the electric potential at a distance r from a charge q. [Punjab 99C]

3. Draw a plot showing the variation of (*i*) electric field (E) and (*ii*) electric potential (V) with distance r due to a point charge Q. [CBSE D 12]

4. Derive an expression for the electric potential due to an electric dipole. [Haryana 01]

5. Derive an expression for the electric potential at a point along the axis line of the dipole. [CBSE D 2000, 08 ; OD 01C, 02, 13C]

6. Show mathematically that the electric potential at any equatorial point of an electric dipole is zero. [CBSE OD 01]

7. Give three differences between the nature of electric potentials of a single point charge and an electric dipole.

8. Obtain an expression for the electric potential at a point due to group of N point charges.

9. Obtain an expression for the potential at a point due to a continuous charge distribution. [CBSE OD 92C]
10. Show that the electric field at any point is equal to the negative of the potential gradient at that point.
11. Describe how can we determine the electric potential at a point from the knowledge of electric field.
12. Two closely spaced equipotential surfaces A and B with potentials V and $V+\delta V$, (where δV is the change in V), are kept δl distance apart as shown in the figure. Deduce the relation between the electric

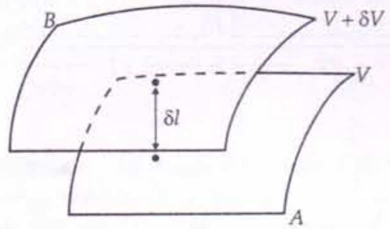


Fig. 2.204

field and the potential gradient between them. Write the two important conclusions concerning the relation between the electric field and electric potentials. [CBSE D 14C]
13. Show that the amount of work done in moving a test charge over an equipotential surface is zero. [Haryana 97]
14. Show that the direction of the electric field is normal to the equipotential surface at every point.
15. Show that the equipotential surfaces are closed together in the regions of strong field and far apart in the region of weak field.
16. Sketch equipotential surfaces for
    (*i*) a positive point charge. [CBSE D 2000]
    (*ii*) a negative point charge. [CBSE D 01]
    (*iii*) two equal and opposite charges separated by a small distance.
    (*iv*) two equal and positive charges separated by a small distance.
    (*v*) a uniform electric field. [CBSE D 2000, 01]
17. (*a*) Draw equipotential surfaces due to a point charge $Q>0$.
    (*b*) Are these surfaces equidistant from each other ? If not, explain why. [CBSE D 11C]
18. Draw the equipotential surfaces due to an electric dipole. Locate the points where the potential due to the dipole is zero. [CBSE OD 13]
19. Two point charges q_1 and q_2 are located at $\vec{r_1}$ and $\vec{r_2}$ respectively in an external electric field $\vec{E}$. Obtain the expression for the total work done in assembling this configuration. [CBSE D 14C]
20. (*a*) Depict the equipotential surfaces for a system of two identical positive point charges placed a distance '*d*' apart.
    (*b*) Deduce the expression for the potential energy of a system of two point charges q_1 and q_2 brought from infinity to the points $\vec{r_1}$ and $\vec{r_2}$ respectively in the presence of external electric field $\vec{E}$. [CBSE D 10 ; OD 15]
21. Derive an expression for the potential energy of an electric dipole of dipole moment $\vec{p}$ in an electric field $\vec{E}$. [Himachal 02 ; CBSE D 08]
22. An electric dipole is held in a uniform electric field $\vec{E}$.
    (*a*) Show that the net force acting on it is zero.
    (*b*) The dipole is aligned with its dipole moment $\vec{p}$ parallel to the field $\vec{E}$. Find :
    (*i*) the work done in turning the dipole till its dipole moment points in the direction opposite to $\vec{E}$.
    (*ii*) the orientation of the dipole for which the torque acting on it becomes maximum. [CBSE OD 12, 14C]
23. Using Gauss's law, show that electric field inside a conductor is zero. [CBSE D 2000]
24. Just outside a conductor electric field is perpendicular to the surface. Give reason.
25. Show that the excess charge on a conductor resides only on its surface.
26. Show that the electric field at the surface of a charged conductor is given by $\vec{E}=\frac{\sigma}{\varepsilon_0}\hat{n}$, where σ is the surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction. [CBSE OD 10]

Or

Derive an expression for the electric field at the surface of a charged conductor. [CBSE OD 09]
27. Show that electric field is zero in the cavity of hollow charged conductor.
28. What is electrostatic shielding ? Mention its two applications.
29. Define electrical capacitance of a conductor. On what factors does it depend ?
30. Show that the capacitance of a spherical conductor is proportional to its radius. Hence justify that farad is a large unit of capacitance. [Himachal 96]

31. An isolated conductor cannot have a large capacitance. Why ?

32. Why does the capacitance of a conductor increase, when an earth connected conductor is brought near it ? Briefly explain.

33. What is a capacitor ? Explain its principle. [Punjab 2000, 02, 03]

34. Derive an expression for the capacitance of a parallel plate capacitor. [CBSE D 05C, 14 ; OD 03]

35. (*a*) Obtain the expression for the energy stored per unit volume in a charged parallel plate capacitor.

(*b*) The electric field inside a parallel plate capacitor is *E*. Find the amount of work done in moving a charge *q* over a closed rectangular loop *a b c d a*. [CBSE D 14]

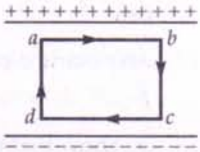


Fig. 2.205

36. Distinguish between polar and non-polar dielectrics. Give one example of each.

37. Three capacitors of capacitances C_1, C_2 and C_3 are connected in series. Find their equivalent capacitance. [CBSE D 92, 93 ; Haryana 94 ; Himachal 97]

38. Three capacitors of capacitances C_1, C_2 and C_3 are connected in parallel. Find their equivalent capacitance. [CBSE D 92, 94 ; Himachal 99]

39. Deduce the expression for the electrostatic energy stored in a capacitor of capacitance 'C' and having charge 'Q'.

How will the (*i*) energy stored and (*ii*) the electric field inside the capacitor be affected when it is completely filled with a dielectric material of dielectric constant 'κ' ? [CBSE OD 08, 12]

40. If two charged conductors are touched mutually and then separated, prove that the charges on them will be divided in the ratio of their capacitances.

41. Two capacitors with capacity C_1 and C_2 are charged to potential V_1 and V_2 respectively and then connected in parallel. Calculate the common potential across the combination, the charge on each capacitor, the electrostatic energy stored in the system and the change in the electrostatic energy from its initial value. [CBSE Sample Paper 08]

42. Explain why the polarization of a dielectric reduces the electric field inside the dielectric. Hence define dielectric constant. [CBSE D 99]

43. Define 'dielectric constant' of a medium. Briefly explain why the capacitance of a parallel plate capacitor increases, on introducing a dielectric medium between the plates. [CBSE OD 06C]

44. What is meant by dielectric polarisation ? Hence establish the relation : $\kappa = 1 + \chi$ [Haryana 01]

45. Two dielectric slabs of dielectric constants κ_1 and κ_2 are filled in between the two plates, each of area A, of the parallel plate capacitor as shown in Fig. 2.206. Find the net capacitance of the capacitor. [CBSE OD 05]

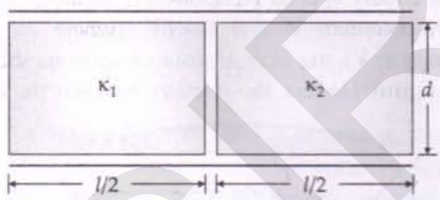


Fig. 2.206

46. A parallel plate capacitor of capacitance C is charged to a potential V. It is then connected to another uncharged capacitor having the same capacitance. Find out the ratio of the energy stored in the combined system to that stored initially in the single capacitor. [CBSE OD 14]

47. If two similar plates, each of area A having surface charge densities $+\sigma$ and $-\sigma$ are separated by a distance d in air, write expressions for : (*i*) The electric field between the two plates (*ii*) The potential difference between the plates (*iii*) The capacitance of the capacitor so formed. [CBSE OD 07]

48. Explain, using suitable diagrams, the difference in the behaviour of a (*i*) conductor and (*ii*) dielectric in the presence of external electric field. Define the terms polarization of a dielectric and write its relation with susceptibility. [CBSE D 15]

49. A capacitor is charged with a battery and then its plate separation is increased without disconnecting the battery. What will be the change in

(*a*) charge stored in the capacitor ?

(*b*) energy stored in the capacitor ?

(*c*) potential difference across the plates of the capacitor ?

(*d*) electric field between the plates of the capacitor ? [CBSE Sample Paper 2011]

50. The charges $q_1 = 3\,\mu F$, $q_2 = 4\,\mu F$ and $q_3 = -7\,\mu F$ are placed on the circumference of a circle of radius 1.0 m, as shown in Fig. 2.207. What is the value of charge q_4 placed on the same circle if the potential at centre, $V_C = 0$? [ISCE 03]

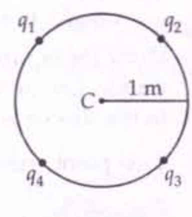


Fig. 2.207

51. Two thin concentric shells of radii r_1 and r_2 $(r_2 > r_1)$ have charges q_1 and q_2. Write the expression for the potential at the surface of inner and outer shells. [CBSE OD 13 C]

52. (a) A charge $+Q$ is placed on a large spherical conducting shell of radius R. Another small conducting sphere of radius r carrying charge 'q' is introduced inside the large shell and is placed at its centre. Find the potential difference between two points, one lying on the sphere and the other on the shell.

(b) How would the charge between the two flow, if they are connected by a conducting wire ? Name the device which works on this fact. [CBSE OD 09]

53. Briefly describe discharging action of sharp points (or corona discharge).

54. Draw a labelled schematic diagram of a Van-de-Graaff generator. State its working principle. Describe briefly how it is used to generate high voltages. [CBSE D 13 C]

Answers

1. Refer to points 3 and 11 of Glimpses.
2. Refer answer to Q. 2 on page 2.2.
3. See Fig. 2.3 on page 2.2.
4. Refer answer to Q. 5 on page 2.3.
5. Refer answer to Q. 3 on page 2.2.
6. Refer answer to Q. 4 on page 2.3.
7. Refer answer to Q. 6 on page 2.3.
8. Refer answer to Q. 7 on page 2.4.
9. Refer answer to Q. 8 on page 2.4.
10. Refer answer to Q. 10 on page 2.11.
11. Refer answer to Q. 11 on page 2.12.
12. Work done in moving a unit positive charge through distance δl,

$$E \times \delta l = V_A - V_B = V - (V + \delta V) = -\delta V$$

$$\therefore \quad E = -\frac{\delta V}{\delta l}$$

For conclusions, refer answer to Q. 10 on page 2.12.

13. Refer answer to Q. 14 on page 2.14.
14. Refer answer to Q. 14 on page 2.14.
15. Refer answer to Q. 14 on page 2.14.
16. Refer answer to Q. 15 on page 2.15.
17. (a) See Fig. 2.25 on page 2.15.

(b) As $E = -\frac{dV}{dr}$ or $dr = -\frac{dV}{E}$

$\therefore$ For constant dV, $dr \propto \frac{1}{E} \propto r^2$

Hence the spacing between the equipotential surface will increase with the increase in distance from the point charge.

18. See Fig. 2.26 on page 2.15. The electric potential is zero at the equatorial points of the dipole.
19. Refer answer to Q. 20 on page 2.17.
20. (a) See Fig. 2.27 on page 2.16.

(b) Refer answer to Q. 20 on page 2.17.

21. Refer answer to Q. 22 on page 2.18.
22. (a) Refer answer to Q. 40 on page 1.41 of chapter 1.

(b) (i) $W = \int_0^\pi \tau d\theta = \int_0^\pi pE \sin\theta = pE[-\cos\theta]_0^\pi = -2pE.$

(ii) As $\tau = pE\sin\theta$, so τ is maximum when $\theta = 90°$.

23. Refer answer to Q. 25 on page 2.24.
24. Refer answer to Q. 25 on page 2.24.
25. Refer answer to Q. 25 on page 2.24.
26. Refer answer to Q. 25 on page 2.24.
27. Refer answer to Q. 25 on page 2.24.
28. Refer answer to Q. 26 on page 2.25.
29. Refer answer to Q. 27 on page 2.26.
30. Refer answer to Q. 29 on page 2.26.
31. Refer answer to Q. 30 on page 2.28.
32. Refer answer to Q. 31 on page 2.28.
33. Refer answer to Q. 31 on page 2.28.
34. Refer answer to Q. 33 on page 2.29.
35. (a) Refer answer to Q. 40 on page 2.49.

(b) $E \perp ab$ and $E \perp dc$, so $W_{ab} = 0$ and $W_{cd} = 0$.

Also, $W_{bc} = -W_{da}$

Total work done in moving charge q over the closed loop $abcda$,

$$W = W_{ab} + W_{bc} + W_{cd} + W_{da}$$
$$= 0 - W_{da} + 0 + W_{da} = 0.$$

36. Refer answer to Q. 43 on page 2.56.
37. Refer answer to Q. 36 on page 2.33.
38. Refer answer to Q. 37 on page 2.33.
39. Refer answer to Q. 38 page 2.48.

When the capacitor is completely filled with a dielectric material and for constant charge Q,

$$C = \kappa C_0 \text{ and } V = V_0 / \kappa$$

(i) $U = \frac{1}{2}CV^2 = \frac{1}{2}(\kappa C_0)\left(\frac{V_0}{\kappa}\right)^2 = \frac{1}{\kappa}\cdot\frac{1}{2}C_0V_0^2 = \frac{U_0}{\kappa}$

(ii) $E = \frac{E_0}{\kappa}$

40. Refer answer to Q. 41 on page 2.49.

41. Refer answer to Q. 42 on page 2.49.

42. Refer answer to Q. 45 on page 2.57.

43. Refer to point 35 of Glimpses on page 2.116.

44. Refer answer to Q. 47 on page 2.58.

45. The given arrangement is equivalent to a parallel combination of two capacitors each with area $A/2$ and plate separation d. Hence the net capacitance of the resulting capacitor is

$$C = C_1 + C_2$$

$$= \frac{\varepsilon_0 (A/2)\kappa_1}{d} + \frac{\varepsilon_0 (A/2)\kappa_2}{d} = \frac{\varepsilon_0 A(\kappa_1 + \kappa_2)}{d}.$$

46. Initial energy stored in the single capacitor

$$= \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C}$$

Capacitance of the combined (parallel) system

$$= C + C = 2C$$

As the total charge q remains the same, so

Final energy stored in the combined system $= \frac{1}{2}\frac{q^2}{2C}$

$\therefore \quad \frac{\text{Final energy}}{\text{Initial energy}} = \frac{1}{2} = \mathbf{1:2}.$

47. (i) Electric field at points between the two plates,

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}.$$

(ii) Potential difference between the plates,

$$V = Ed = \frac{\sigma d}{\varepsilon_0}.$$

(iii) Capacitance of the capacitor so formed,

$$C = \frac{q}{V} = \frac{\sigma A}{\sigma d/\varepsilon_0} = \frac{\varepsilon_0 A}{d}.$$

48. Refer answer to Q. 43 on page 2.56.

Polarisation of a dielectric. The induced dipole moment set up per unit volume of a dielectric when placed in an external electric field is called polarisation. For linear isotropic dielectrics,

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

where χ_e is the electric susceptibility of the dielectric medium.

49. $C = \frac{\varepsilon_0 A}{d}$.

When d is increased, C decreases.

(a) $q = CV$ *decreases* due to the decrease in the value of C.

(b) $U = \frac{1}{2}CV^2$ *decreases* due to the decrease in the value of C.

(c) V remains *unchanged* because the battery remains connected.

(d) $E = V/d$ *decreases* due to the increase in the value of d.

50. As $V_C = 0$

$\therefore \quad \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right] = 0$

or $\quad q_1 + q_2 + q_3 + q_4 = 0$

or $\quad 3 + 4 - 7 + q_4 = 0$

or $\quad \mathbf{q_4 = 0.}$

51. Potential at the surface of inner shell,

V_1 = Potential due to its own charge q_1
+ Potential due to charge q_2 on outer shell

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

Potential at the surface of outer shell

V_2 = Potential due to charge q_1 on inner shell
+ Potential due to charge q_2 on outer shell

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_2} + \frac{q_2}{r_2}\right)$$

52. Refer answer to Q. 55 on page 2.67.

53. Refer answer to Q. 54 on page 2.67.

54. Refer answer to Q. 56 on page 2.67.

TYPE C : LONG ANSWER QUESTIONS (5 marks each)

1. Find the expression for the electric field intensity and the electric potential, due to a dipole at a point on the equatorial line. Would the electric field be necessarily zero at a point where the electric potential is zero ? Give an example to illustrate your answer. [CBSE Sample Paper 2011]

2. Define electrostatic potential energy of a charge system. Derive an expression for the potential energy of a system of three point charges. Generalise the result for a system of N point charges.

3. Define the term electric dipole moment. Derive an expression for the total work done in rotating the dipole through an angle θ in uniform electric field $\vec{E}$. [CBSE OD 93, 95 ; D 96C]
4. Derive an expression for the potential energy of an electric dipole placed in a uniform electric field. Hence discuss the conditions of its stable and unstable equilibrium.
5. Explain the principle of a capacitor. Derive an expression for the capacitance of a parallel plate capacitor. [CBSE D 92, 94]
6. Obtain the expression for the capacitance of a parallel plate capacitor.

   Three capacitors of capacitances C_1, C_2 and C_3 are connected (*i*) in series, (*ii*) in parallel. Show that the energy stored in the series combination is the same as that in the parallel combination. [CBSE OD 03]
7. Deduce an expression for the total energy stored in a parallel plate capacitor and relate it to the electric field. [CBSE F 94C]
8. (*a*) Derive the expression for the energy stored in a parallel plate capacitor. Hence obtain the expression for the energy density of the electric field.

   (*b*) A fully charged parallel plate capacitor is connected across an uncharged identical capacitor. Show that the energy stored in the combination is less than that stored initially in the single capacitor. [CBSE OD 15]
9. Define the terms (*i*) capacitance of a capacitor (*ii*) dielectric strength of a dielectric. When a dielectric is inserted between the plates of a charged parallel plate capacitor, fully occupying the intervening region, how does the polarization of the dielectric medium affect the net electric field ? For linear dielectrics, show that the introduction of a dielectric increases its capacitance by a factor κ, characteristic of the dielectric. [CBSE D 08C]
10. Find the expression for the capacitance of a parallel plate capacitor of area A and plate separation d if (*i*) a dielectric slab of thickness t and (*ii*) a metallic slab of thickness t, where ($t < d$) are introduced one by one between the plates of the capacitor. In which case would the capacitance be more and why ? [CBSE Sample Paper 2011]
11. What is a dielectric ? A dielectric slab of thickness t is kept between the plates of a parallel plate capacitor separated by distance d. Derive the expression for the capacitance of the capacitor for $t << d$. [Himachal 02 ; CBSE D 93 ; OD 01C]
12. (*a*) Show that in a parallel plate capacitor, if the medium between the plates of a capacitor is filled with an insulating substance of dielectric constant κ, its capacitance increases. (*b*) Deduce the expression for the energy stored in a capacitor of capacitance C with charge Q. [CBSE D 09C]
13. (*a*) A small sphere, of radius 'a', carrying a positive charge q, is placed concentrically inside a larger hollow conducting shell of radius $b (b > a)$. This outer shell has a charge Q on it. Show that if these spheres are connected by a conducting wire, charge will always flow from the inner sphere to the outer sphere, irrespective of the magnitude of the two charges. [CBSE F 15]

    (*b*) Name the machine which makes use of this principle. Draw a simple labelled line diagram of this machine. What 'practical difficulty' puts on upper limit on the maximum potential difference which this machine can built up ? [CBSE D 09C ; OD 14]
14. Explain the principle of a device that can build up high voltages of the order of a few million volts.

    Draw a schematic diagram and explain the working of this device.

    Is there any restriction on the upper limit of the high voltages set up in this machine ? Explain. [CBSE D 12]

Answers

1. For derivation of electric field intensity at equatorial point of a dipole, refer answer to Q. 38 on page 1.40 of chapter 1.

$$E_{equa} = \frac{1}{4\pi\varepsilon_0} \frac{p}{(r^2 + a^2)^{3/2}}$$

For derivation of electric potential at an equatorial point of a dipole, refer answer to Q. 4 on page 2.3 of chapter 2.

$$V_{equa} = 0$$

No, the electric field may not be necessarily zero at a point where the electric potential is zero. *For example*, the electric potential at an equatorial point of a dipole is zero, while electric field is not zero.

2. Refer answer to Q. 18 on page 2.16.
3. Refer answer to Q. 22 on page 2.18.
4. Refer answer to Q. 22 on page 2.18.
5. Refer answer to Q. 31 on page 2.28 and Q. 33 on page 2.29.

6. Refer answer to Q. 33 on page 2.29 and Q. 39 on page 2.48.
7. Refer answer to Q. 38 on page 2.48 and Q. 40 on page 2.48.
8. (a) Refer answer to Q. 38 on page 2.48 and Q. 40 on page 2.48.
(b) Refer to the solution of Example 75 on page 2.52.
9. Refer answer to Q. 45 on page 2.57 and Q. 49 on page 2.59.
10. For derivation, refer answers to Q. 49 on page 2.59 and Q. 50 on page 2.60.

$$C_{dielectric} = \frac{\varepsilon_0 A}{d - t + \frac{t}{\kappa}}$$

$$C_{metal} = \frac{\varepsilon_0 A}{d - t} \quad \text{[For metal, } \kappa = \infty\text{]}$$

Clearly, $C_{metal} > C_{dielectric}$

11. Refer answer to Q. 49 on page 2.59.
12. (a) Refer answer to Q. 49 on page 2.59.
(b) Refer answer to Q. 38 on page 2.48.
13. (a) Refer answer to Q. 55 on page 2.67
(b) A Van-de-Graaff generator works on this principle. See Fig. 2.130. The potential on the outer surface of its metallic shell cannot exceed the breakdown field of air $(\simeq 3 \times 10^6 \text{Vm}^{-1})$ because then the charges begin to leak into air. This puts the limit on the potential difference which the machine can built up.
14. The device is Van de Graff generator. For its principle and working, refer answer to Q. 56 on page 2.67.
Yes, there is a restriction on the upper limit of the high voltages set up in the Van de Graaff generator. The high voltages can be built up only upto the breakdown field of the surrounding medium.

TYPE D : VALUE BASED QUESTIONS (4 marks each)

1. Immediately after school hour, as Bimla with her friends came out, they noticed that there was a sudden thunderstorm accompanied by the lightning. They could not find any suitable place for shelter. Dr. Kapoor who was passing thereby in his car noticed these children and offered them to come in his car. He even took care to drop them to the locality where they were staying. Bimla's parents, who were waiting, saw this and expressed their gratitude to Dr. Kapoor. [CBSE OD 15C]
(a) What values did Dr. Kapoor and Bimla's parents display ?
(b) Why is it considered safe to be inside a car especially during lightning and thunderstorm ?
(c) Define the term 'dielectric strength'. What does this term signify ?

2. One evening, Pankaj outside his house fixed a two metre high insulating slab and attached a large aluminium sheet of area $1 m^2$ over its top. To his surprise, next morning when he incidently touched the aluminium sheet, he received an electric shock. He got afraid. He narrated the incident to his Physics teacher in the school who explained him the reason behind it.
Answer the following questions based on the above information :
(a) What are the values being displayed by Pankaj ?
(b) What may be the reason behind the electric shock received by Pankaj ?

Answers

1. (a) Dr. Kapoor displayed helpfulness, empathy and scientific temper.
Bimla parents displayed gratefulness and indebtedness.
(b) It is safer to sit inside a car during a thunderstorm because the metallic body of the car becomes an electrostatic shielding from lightning.
(c) The maximum electric field that a dielectric medium can withstand without break-down of its insulating property is called its dielectric strength. It signifies the maximum electric field upto which the dielectric can safely play its role.
2. (a) Keen observer and curiosity.
(b) The aluminium sheet and the ground form a capacitor alongwith the insulating slab. The discharging current of the atmosphere charges the capacitor steadily and raises its voltage. So, when Pankaj touches the aluminium sheet, he receives an electric shock.

COMPETITION SECTION

Electrostatic Potential and Capacitance

GLIMPSES

1. Potential difference. The potential difference between two points is defined as the amount of work done in bringing a unit positive charge from one point to another against the electrostatic forces.

$$\text{Potential difference} = \frac{\text{Work done}}{\text{Charge}}$$

or $$V_{AB} = V_B - V_A = \frac{W_{AB}}{q}$$

2. SI unit of potential difference is volt (V). The potential difference between two points in an electric field is said to be 1 volt if 1 joule of work has to be done in moving a positive charge of 1 coulomb from one point to the other against the electrostatic forces.

$$1\,V = 1\,JC^{-1} = 1\,Nm\,C^{-1}$$

3. Electric potential. It is defined as the amount of work done in bringing a unit positive charge from infinity to the observation point against the electrostatic forces.

$$\text{Electric potential} = \frac{\text{Work done}}{\text{Charge}}$$

or $$V = \frac{W}{q}$$

Electric potential is a scalar quantity.

4. SI unit of electric potential is volt. The electric potential at a point in an electric field is said to be 1 volt if one joule of work has to be done in moving a positive charge of 1 coulomb from infinity to that point against the electrostatic forces.

5. Electric potential due to a point charge. The electric potential of a point charge q at distance r from it is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} \qquad \textit{i.e.,} \quad V \propto \frac{1}{r}$$

It is spherically symmetric.

6. Electric potential due to a dipole. Electric potential at a point having position vector $\vec{r}$, due to a dipole of moment $\vec{p}$ at the origin is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{p} \cdot \vec{r}}{r^3} = \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2}$$

At points on the axial line of the dipole ($\theta = 0°$ or $180°$),

$$V_{axial} = \pm \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^2}$$

At points on the equatorial line of the dipole ($\theta = 90°$),

$$V_{equa} = 0.$$

7. Electric potential due to a group of N point charges. If $r_1, r_2, r_3 \ldots r_N$ are the distances of N point charges from the observation point, then

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots + \frac{q_N}{r_N}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i}$$

8. Determination of electric field from electric potential. The rate of change of potential with distance is called potential gradient. Electric

field at any point is equal to the negative of the potential gradient at that point

$$E = -\frac{dV}{dr}$$

$\therefore$ SI unit of electric field = Vm^{-1}

The direction of $\vec{E}$ is in the direction of steepest decrease of potential.

9. **Determination of electric potential from electric field.** The electric potential at a point having position vector $\vec{r}$ is given by

$$V = -\int_{\infty}^{r} \vec{E} \cdot \vec{dr}$$

10. **Equipotential surface.** Any surface that has same electric potential at every point on it is called an equipotential surface. The surface of a charged conductor is an equipotential surface. Some of the important properties of equipotential surface are as follows :
    (*i*) No work is done in moving a test charge over an equipotential surface.
    (*ii*) Electric field is always normal to the equipotential surface at every point.
    (*iii*) Equipotential surfaces are close together in the regions of strong field and farther apart in the regions of weak field.
    (*iv*) No two equipotential surfaces can intersect each other.

11. **Electric potential energy.** The electric potential energy of a system of point charges is defined as the amount of work done in assembling the charges at their locations by bringing them in, from infinity.

    P.E. of a charge = Charge × Electric potential at the given point

    It is measured in joule (J) or electron volt (eV).

    $$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

12. **Potential energy of a system of two point charges.** If two point charges q_1 and q_2 are separated by distance r_{12}, then their potential energy is

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$

13. **Potential energy of a system of three point charges.** It is given by

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_3 q_1}{r_{31}}\right]$$

14. **Potential energy of N point charges.** It is given by

$$U = \frac{1}{4\pi\varepsilon_0} \sum_{\text{All pairs}} \frac{q_i q_j}{r_{ij}}$$

$$= \frac{1}{2} \cdot \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} \frac{q_i q_j}{r_{ij}}$$

15. **Potential energy of a dipole in a uniform electric field.** It is equal to the amount of work done in turning the dipole from orientation θ_1 to θ_2 in the field E.

$$U = -pE(\cos\theta_2 - \cos\theta_1)$$

If initially the dipole is perpendicular to the field E, $\theta_1 = 90°$ and $\theta_2 = \theta$ (say), then

$$U = -pE\cos\theta = -\vec{p} \cdot \vec{E}$$

When $\theta = 0°$, $U = -pE$ *i.e.*, the potential energy of the dipole is minimum. The dipole is in *stable equilibrium*.

When $\theta = 90°$, $U = 0$

When $\theta = 180°$, $U = +pE$

i.e., the potential energy of the dipole is maximum. The dipole is in *unstable equilibrium*.

16. **Conductors and insulators.** Conductors are the substances which allow large scale physical movement of electric charges through them when an external electric field is applied. They contain a large number of free electrons. Insulators are the substances which do not allow physical movement of electric charges through them when an external electric field is applied. They contain a negligibly small number of free charge carriers.

17. **Electrostatic properties of a conductor.** When placed in an electrostatic field, a conductor shows the following properties :
    (*i*) Net electrostatic field is zero in the interior of a conductor.
    (*ii*) Just outside the surface of a conductor, electric field is normal to the surface.

(*iii*) The net charge in the interior of a conductor is zero and any excess charge resides on its surface.

(*iv*) Potential is constant within and on the surface of a conductor.

(*v*) Electric field at the surface of a charged conductor is proportional to the surface charge density.

(*vi*) Electric field is zero in the cavity of a hollow charged conductor.

18. Electrostatic shielding. The phenomenon of making a region free from any electric field is called electrostatic shielding. It is based on the fact that electric field vanishes inside the cavity of a hollow conductor.

19. Capacitance of a conductor. It is the charge required to increase the potential of a conductor by unit amount.

$$\text{Capacitance} = \frac{\text{Charge}}{\text{Potential}}$$

or
$$C = \frac{q}{V}$$

20. Capacitance of a spherical conductor. It is proportional to the radius R of the spherical conductor.

$$C = 4\pi\varepsilon_0 R$$

21. Capacitor. It is an arrangement of two conductors separated by an insulating medium that is used to store electric charge and electric energy.

22. Capacitance of a capacitor. The capacitance of a capacitor is the charge required to be supplied to one of its conductors so as to increase the potential difference between two conductors by unit amount.

$$C = \frac{q}{V}$$

23. Farad. It is the SI unit of capacitance. The capacitance of a capacitor is 1 farad (F) if 1 coulomb of charge is transferred from its one plate to another on applying a potential difference of 1 volt across the two plates.

$$1 \text{ farad} = \frac{1 \text{ coulomb}}{1 \text{ volt}} \quad \text{or} \quad 1\,\text{F} = \frac{1\,\text{C}}{1\,\text{V}}$$

$$1\,\text{mF} = 10^{-3}\,\text{F},\ 1\,\mu\,\text{F} = 10^{-6}\,\text{F},\ 1\,\text{pF} = 10^{-12}\,\text{F}.$$

24. Parallel plate capacitor. It consists of two large parallel conducting plates, each of area A, and separated by a small distance d. Its capacitance is

$$C = \frac{\varepsilon_0 A}{d}$$

25. Spherical capacitor. It consists of two concentric spherical conducting shells of inner and outer radii a and b.

$$C = \frac{4\pi\varepsilon_0 ab}{b - a}$$

26. Cylindrical capacitor. It consists of two coaxial conducting cylinders of inner and outer radii a and b and of common length l.

$$C = 2\pi\varepsilon_0 \frac{l}{\log_e \frac{b}{a}} = 2\pi\varepsilon_0 \frac{l}{2.303 \log_{10} \frac{b}{a}}$$

27. Capacitors in series. The equivalent capacitance C_s of number of capacitors connected in series is given by

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

In a series combination of capacitors, the charge on each capacitor is same but the potential difference across any capacitor is inversely proportional to its capacitance.

28. Capacitors in parallel. The equivalent capacitance of a number of capacitors connected in parallel is given by

$$C_p = C_1 + C_2 + C_3 + \ldots$$

In a parallel combination of capacitors, the potential difference across each capacitor is same but the charge on each capacitor is proportional to its capacitance.

29. Energy stored in a capacitor. The energy stored in a capacitor of capacitance C and charge q with voltage V is

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \cdot \frac{Q^2}{C} = \frac{1}{2} QV$$

30. Energy density. The electrical energy stored per unit volume or energy density in a region with electric field E is

$$u = \frac{1}{2} \varepsilon_0 E^2$$

31. Common potential. If a number of conductors of capacitances C_1, C_2, C_3, ..., at potentials V_1, V_2, V_3, ..., having charges q_1, q_2, q_3, ... respectively are placed in contact, their common potential V is given by

$$V = \frac{\text{Total charge}}{\text{Total capacitance}} = \frac{q_1 + q_2 + q_3 + ...}{C_1 + C_2 + C_3 + ...}$$

$$= \frac{C_1V_1 + C_2V_2 + C_3V_3 + ...}{C_1 + C_2 + C_3 + ...}.$$

32. Loss of energy on sharing charges. If two conductors of capacitances C_1 and C_2 at potentials V_1 and V_2 respectively are connected together, a loss of energy takes place which is given by

$$\Delta U = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2.$$

33. Dielectric. A dielectric is a substance which does not allow the flow of charges through it but permits them to exert electrostatic forces on one another. It is essentially an insulator which can be polarised through small localised displacements of its charges.

34. Polar and non-polar dielectrics. The dielectrics made of polar molecules (such as HCl, NH_3, H_2O, CH_3OH, etc.) are called polar dielectrics. In a polar molecule, the centre of mass of positive charges does not coincide with the centre of mass of negative charges.

The dielectrics made of non-polar molecules are called non-polar dielectrics. In a non-polar molecule, the centre of mass of positive charges coincides with the centre of mass of negative charges *e.g.*, H_2, O_2, CO_2, CH_4, etc.

35. Polarisation of dielectric. If the medium between the plates of a capacitor is filled with a dielectric, the electric field due to the charged plates induces a net dipole moment in the dielectric. This effect is called polarisation which induces a field in the opposite direction. The net electric field inside the dielectric and hence the potential difference between the plates are reduced. Consequently, the capacitance C increases from its value C_0 when there is vacuum.

$$C = \kappa\, C_0.$$

36. Dielectric constant. It is the ratio of the capacitance (C) of the capacitor with the dielectric as the medium to its capacitance (C_0) when conductors are in vacuum.

$$\kappa = \frac{C}{C_0}$$

It is also equal to the ratio of the applied electric field (E_0) to the reduced value of electric field (E) on inserting the dielectric slab between the plates of the capacitor.

$$\kappa = \frac{E_0}{E} = \frac{E_0}{E_0 - E'}.$$

Here E' is the field set up due to polarisation of the dielectric in the opposite direction of E_0.

37. Capacitance of a parallel plate capacitor filled with a dielectric.

$$C = \kappa\, C_0 = \frac{\varepsilon_0\, \kappa\, A}{d}.$$

38. Capacitance of a parallel plate capacitor with a dielectric slab between its plates. If t is the thickness of the dielectric slab and $t < d$, then

$$C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\kappa}\right)}.$$

39. Capacitance of a parallel plate capacitor with conducting slab between its plates. For $t < d$,

$$C = \left(\frac{d}{d-t}\right)\frac{\varepsilon_0\, A}{d} = \left(\frac{d}{d-t}\right) C_0.$$

40. Capacitance of a spherical capacitor filled with a dielectric.

$$C = 4\,\pi\,\varepsilon_0\,\kappa\,\frac{ab}{b-a}.$$

41. Capacitance of a cylindrical capacitor filled with a dielectric

$$C = \frac{2\,\pi\,\varepsilon_0\,\kappa\, l}{2.303\,\log_{10}\frac{b}{a}}.$$

42. Van de Graaff generator. It is an electrostatic generator capable of building up high potential differences of the order of 10^7 volts. It is based on the principle that when a charged conductor is brought into internal contact with a hollow conductor, it transfers whole of its charge to the hollow conductor, howsoever high the potential of the latter may be. Also, it uses discharging action of sharp points. It is used for accelerating charged particles.